IFT 4031/7031,
Machine Learning for Signal Processing
Week6: Machine Learning 3, Classification

Cem Subakan





UNIVERSITÉ LAVAL



Mila

Admin

- If you have questions on your project just let me know.
  - S'il y a des hesitations laissez nous savoir.
- Did you manage to start using the computational resources?
  - Avez-vous pu commencer à utiliser les ressources de calcul?
- Aujourd'hui: Classification

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Classification Intro

Generative Classification

Discriminative Classification
- Linear Classifiers
- The perceptron algorithm
- Logistic Regression

Non-Linear Classification
- Kernel Logistic Regression
- Neural Network Classification

Evaluating Classifiers

Supervised Learning

- So far we have mostly done unsupervised learning to discover structures.
  - Jusqu'à maintenant on a majoritairement fait de l'apprentissage non-supervisé. Le but était de découvrir la structure.
- Now, we will do classification / detection, or supervised learning in other words.
  - Maintenant on va faire de l'apprentissage supervisé.

A simple example

Query



Classes



A simple example

Query



Classes



How can we assign the query to a class? / Comment peut-on assigner l'exemple en question à une classe?

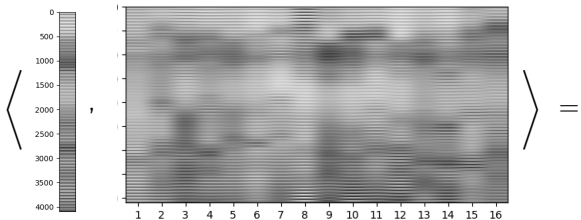
We can simply calculate inner products!

We can calculate inner products! / On peut calculer des produits scalaires!

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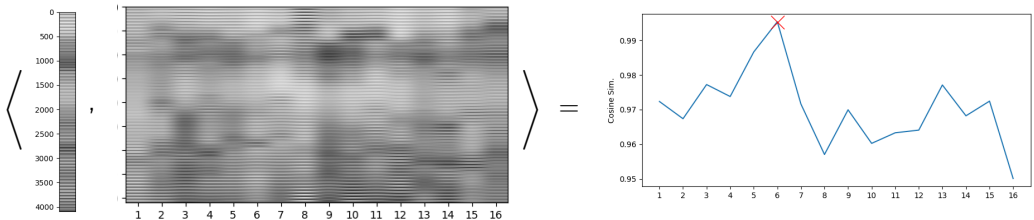
Query Classes



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Query Classes



It worked but will it this time?

Query



Classes



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Classes



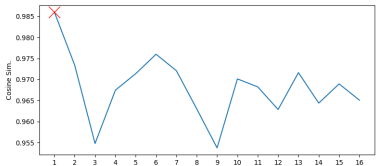
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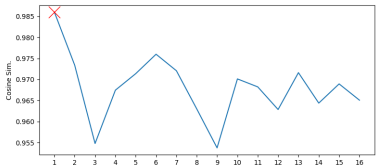
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Query



Classes





Didn't work this time!
Ça n'a pas fonctionné!

What can we do to fix this?

- We will take a statistical approach (as usual). This will consider the class distributions. / On va prendre une approche de statistique comme d'habitude qui va prendre en compte les distributions de classes.

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- Approach 1: Generative Classification
  - Approche 1: Classification Générative
    - We will learn a distribution over samples in the class. / On va fitter une distribution sur les échantillons dans la classe.
- Approach 2: Discriminative Classification
  - Approche 2: Classification Discriminative
    - We will learn how the class distribution separate. / On va fitter une fonction pour comprendre comment les classes se séparent.

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- We will build step by step towards neural nets, and why we need them in this lecture. / On va builder graduellement à pourquoi on a besoin des réseaux de neurones dans ce cours.
  - Linear Classifiers (Perceptron Algo. Logistic Regression, Kernel Methods, and then Multilayer perceptron)

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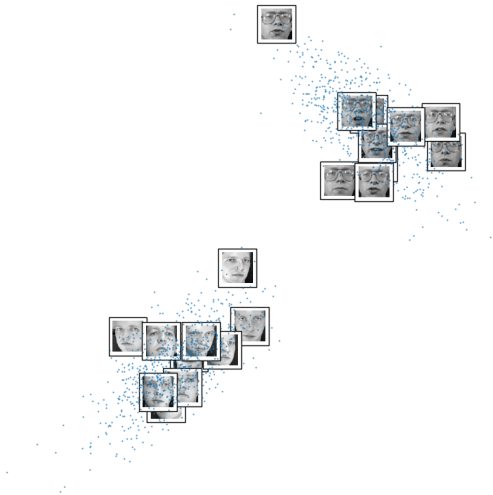
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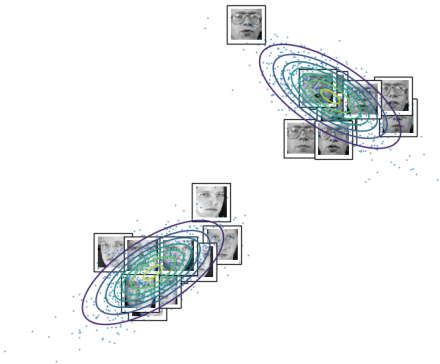
Generative Classification

- Generative Classification fits distributions to each class / Classification générative fit une distribution à chaque classe.



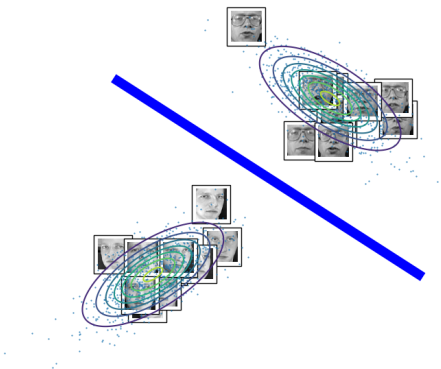
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Generative Classification

- Training Time: Fit a distribution $p(x|\theta_k)$ to each class k with maximum likelihood.
  - L'entrainement: On va fitter une distribution $p(x|\theta_k)$ pour chaque classe k avec maximum likelihood.

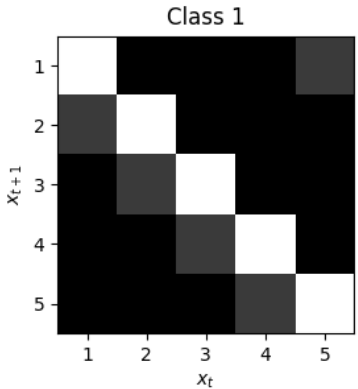
$$\max_{\theta_k} \sum_{n \in \text{class } k} \log p(x_n|\theta_k)$$

- Test Time: Evaluate the likelihood for each model. Assign to the largest likelihood class!
  - Au test: Evaluez le likelihood pour chaque modèle. Assignez à la classe avec le likelihood plus grand.

$$\widehat{c} = \arg\max_k \log p(x_{\text{test}}|\theta_k)$$

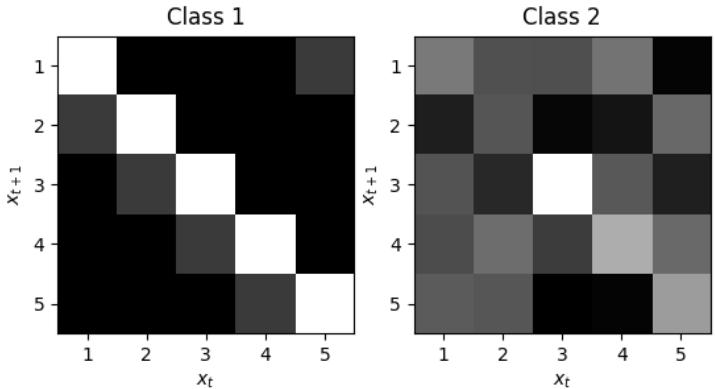
Example Application: Classifying Sequences

- class 1 $\sim$ Markov(A_1)
- class 2 $\sim$ Markov(A_2)



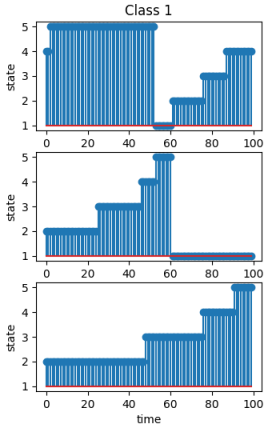
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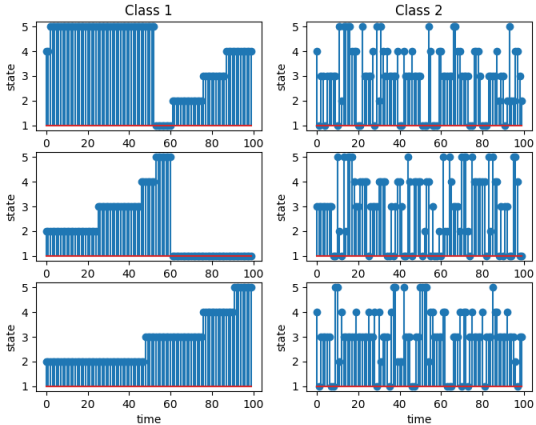
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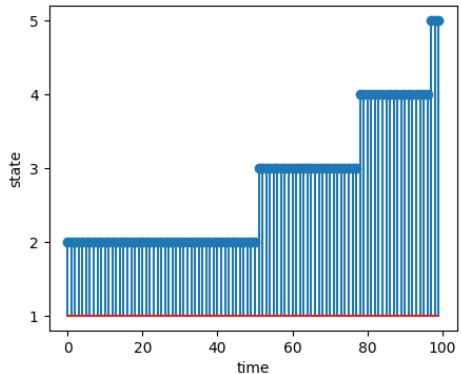
Example Application: Classifying Sequences

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Test time

- Here's a test sequence / Une séquence de test



- Do you think it belongs to which class? / Vous pensez que cette séquence appartient à quelle classe?

Test time

$$\log p(x_{1:T}|A_k, \pi_k) = \log \left(p(x_1|\pi_k) \prod_{t=2}^{T} p(x_t|x_{t-1}, A_k) \right)$$

Test time

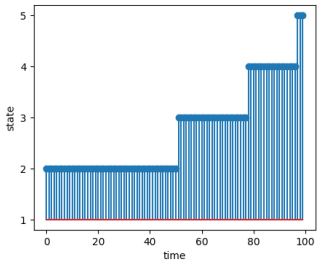
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$$= \log p(x_1|\pi_k) + \sum_{t=2}^{T} \log p(x_t|x_{t-1}, A_k)$$

Test time

$$
\begin{aligned}
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&= \log p(x_1|\pi_k) + \sum_{t=2}^{T} \log p(x_t|x_{t-1}, A_k) \\
&= \log \pi_k + \sum_{t=2}^{T} \log A_k(x_t, x_{t-1})
\end{aligned}
$$

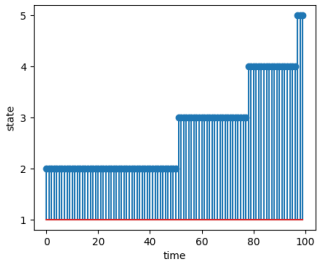
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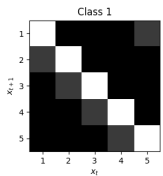
- Observation sequence / Séquence observé



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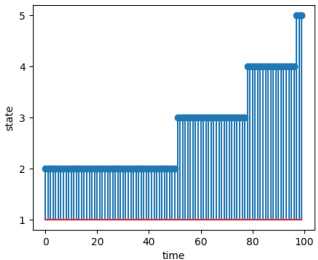




$$\log p(x_{1:T}|A_1, \pi_1) = \log\left(\pi_1 \prod_{t=2}^{T} A_1(x_t, x_{t-1})\right)$$
$$= \log\left(\frac{1}{5} \cdot 0.95 \cdot 0.95 \cdot 0.95 \ldots\right)$$
$$= -15.52$$

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- Observation sequence / Séquence observé

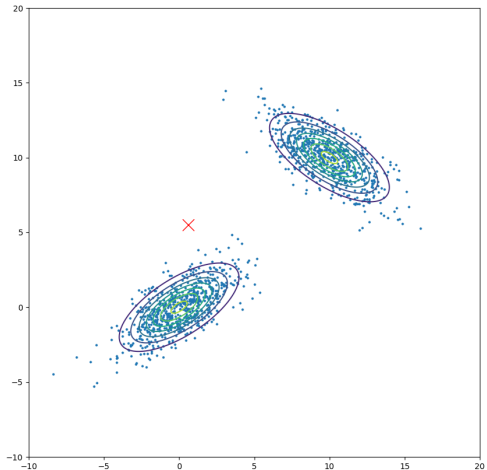


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$$= -15.52$$

$$\log p(x_{1:T}|A_2, \pi_2) = \log\left(\pi_2 \prod_{t=2}^{T} A_2(x_t, x_{t-1})\right)$$
$$= \log\left(\frac{1}{5} \cdot 0.21 \cdot 0.21 \cdot 0.21 \ldots\right)$$
$$= -117.93$$

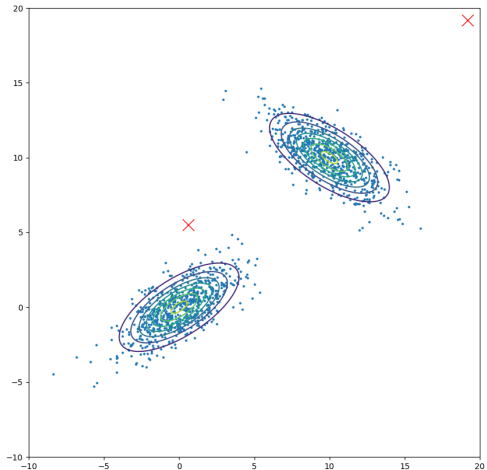
Let's do something simpler

Which class should we assign the red point? / Quel classes doit-on assigner au point rouge?



How about this?

How about this second point? / Et le deuxième point?



The principled way to derive the decision boundaries

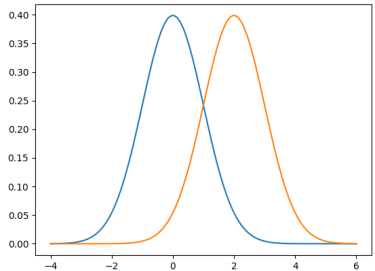
- The discriminant function / La fonctionne de discrimination

$$d(x) = \log \frac{p(x|c=1)}{p(x|c=2)} = \log \frac{p(x|\theta_1)}{p(x|\theta_2)}$$

- The decision boundary: $d(x) = 0$.
- For Gaussians / simple densities this can be derived analytically.

Deriving the decision boundary

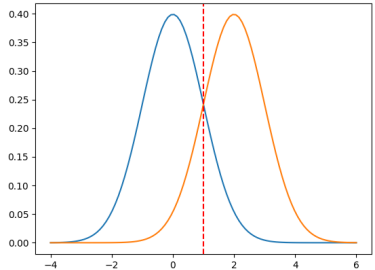
- Derive the boundary in 1d case / Dérivons dans le cas 1d.



$$d(x) = \log \frac{p(x|\theta_1)}{p(x|\theta_2)} = \log \frac{\mathcal{N}(x; 0, 1)}{\mathcal{N}(x; 2, 1)}$$

Deriving the decision boundary

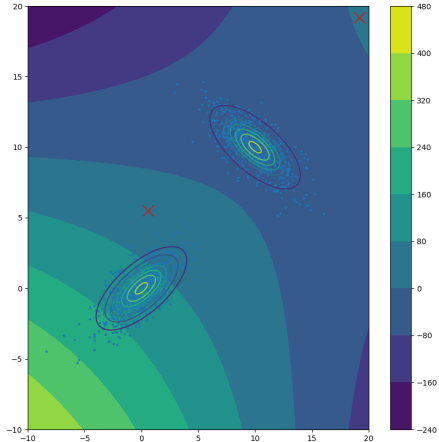
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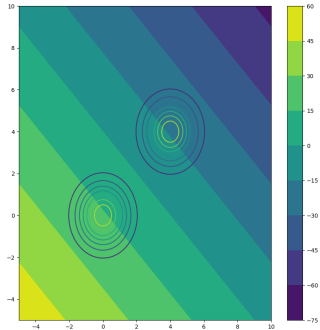
$$
\begin{aligned}
d(x) &= \log \frac{p(x|\theta_1)}{p(x|\theta_2)} = \log \frac{\mathcal{N}(x;0,1)}{\mathcal{N}(x;2,1)} \\
&= -\frac{x^2}{2} + \frac{(x-2)^2}{2} = 0 \\
\rightarrow x^2 &= (x-2)^2 \rightarrow x = 1
\end{aligned}
$$

The 2d decision boundary

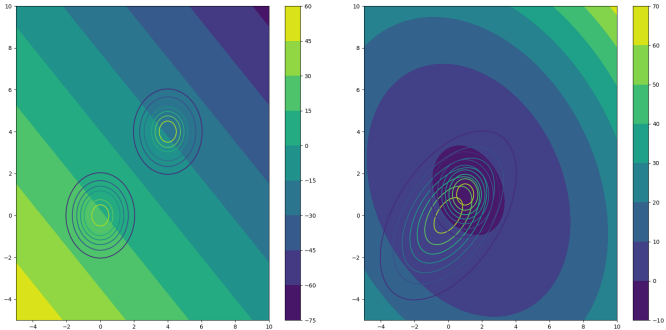
Plot $d(x) = \log \frac{\mathcal{N}([x,y]^\top; \mu_1, \Sigma_1)}{\mathcal{N}([x,y]^\top; \mu_2, \Sigma_2)}$



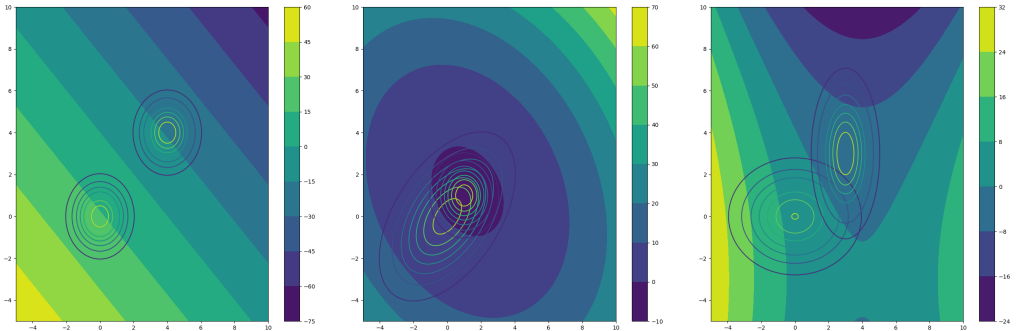
Few other cases



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All these cases can be derived analytically! / C'est possible de les calculer analytiquement

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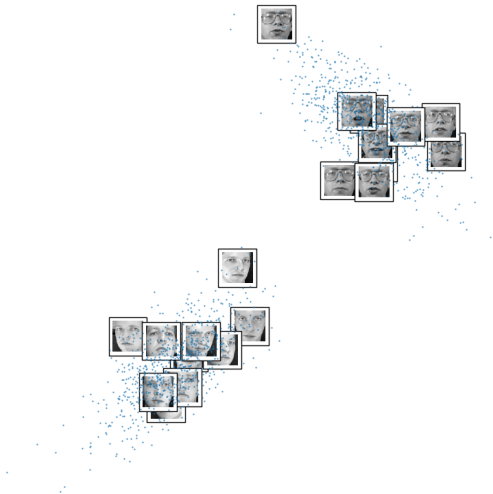
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- Discriminative classification directly learns a decision boundary / Classification discriminative apprend directement un borne de décision.



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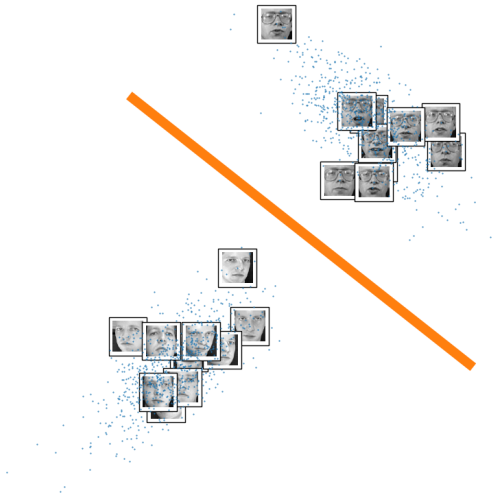


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Linear Classifier

- Let's learn a vector $w \in \mathbb{R}^L$, such that $c \approx w^\top x$. / On va apprendre une vecteur w pour approximer c.

Linear Classifier

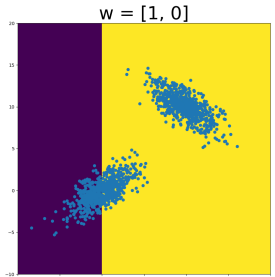
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- More specifically, we want to have (on veut avoir)

$$w^\top x \geq 0 \text{ if } c = 1, \qquad w^\top x \leq 0 \text{ if } c = 0$$

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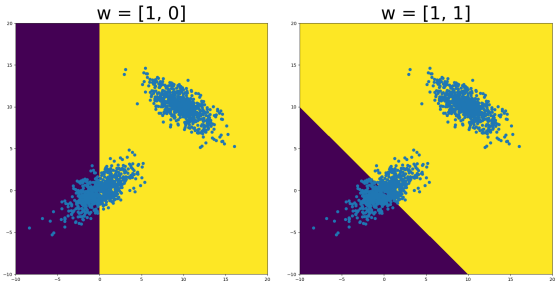
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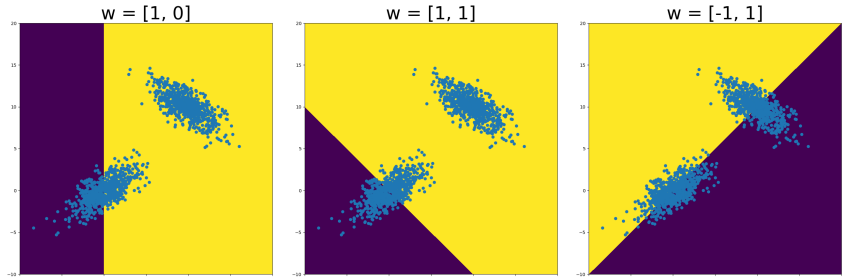
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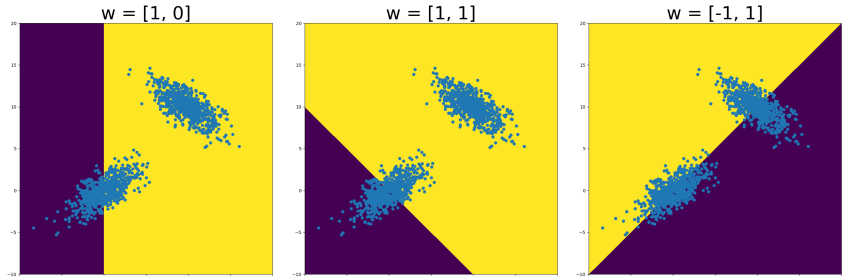
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- Btw, we will also add a bias term so that $f(x) = w^\top x + b$ / En passant on va aussi ajouter un biais.

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  - If $\text{sgn}(w^\top x_n) = -c_n$, then $w = w + \eta c_n x_n$. (η is a learning rate)

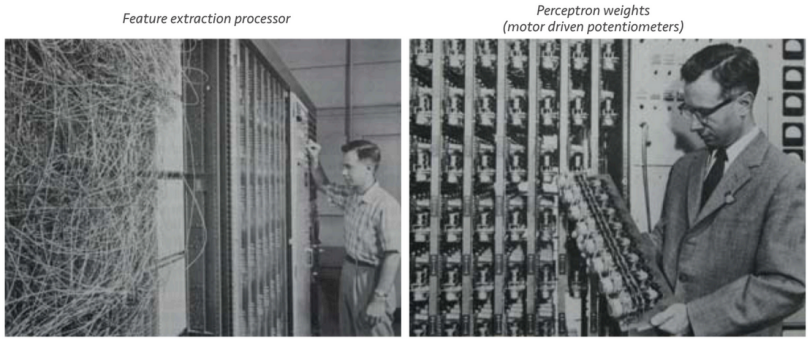
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  - If $\text{sgn}(w^\top x_n) = -c_n$, then $w = w + \eta c_n x_n$. (η is a learning rate)
  - Do these updates until convergence. / On répète jusqu'qu'on converge.

The perceptron algorithm

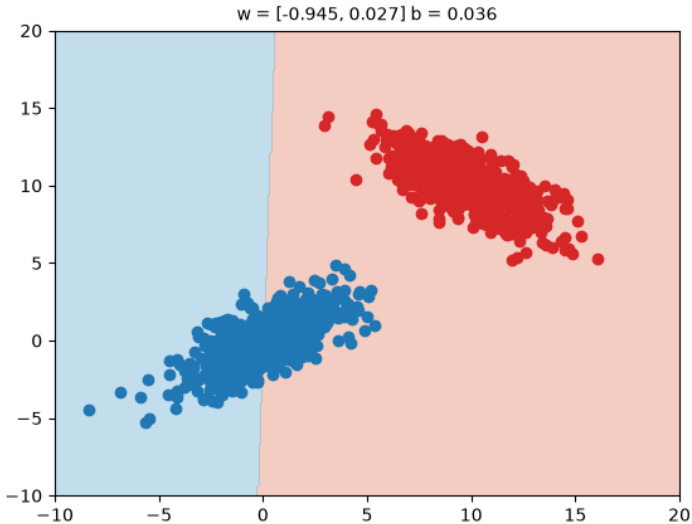
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- The perceptron algorithm.
  - If $\text{sgn}(w^\top x_n) = c_n$ do nothing.
  - If $\text{sgn}(w^\top x_n) = -c_n$, then $w = w + \eta c_n x_n$. (η is a learning rate)
  - Do these updates until convergence. / On répète jusqu'qu'on converge.
  - Note that $c_n \in \{-1, 1\}$. / Notez que $c_n \in \{-1, 1\}$.

The perceptron

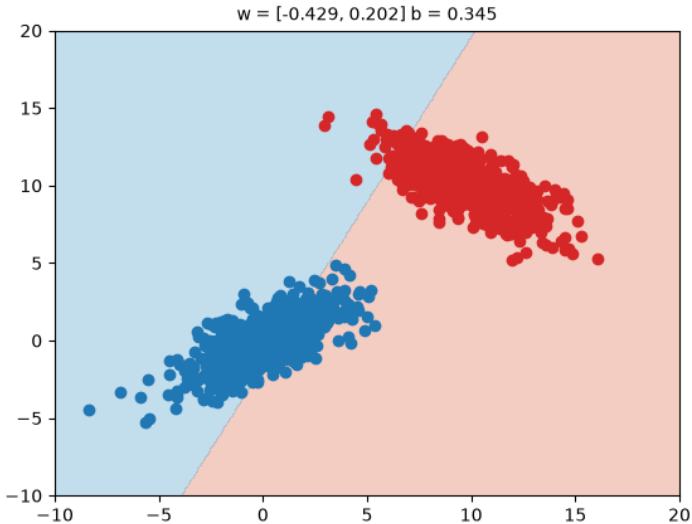


Taken from UIUC MLSP class

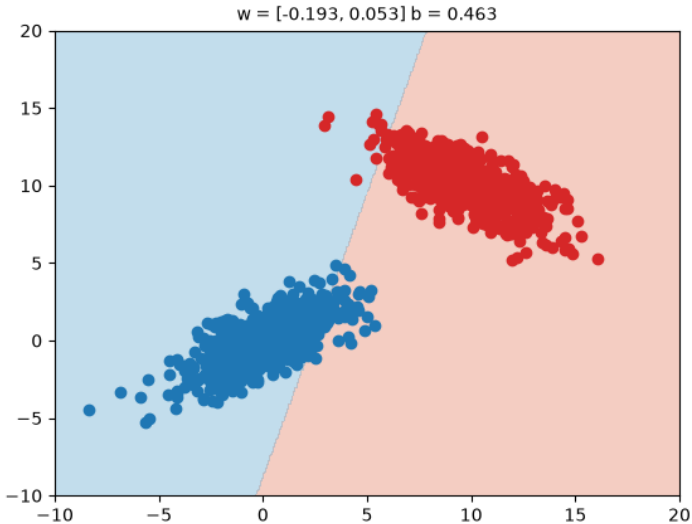
Perceptron Epochs



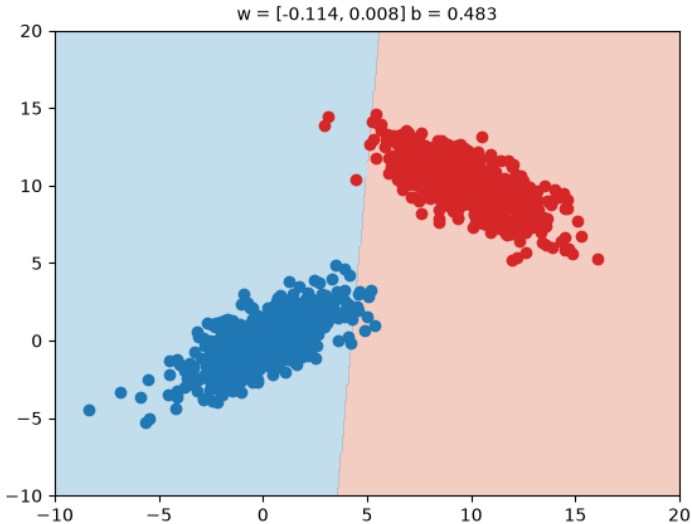
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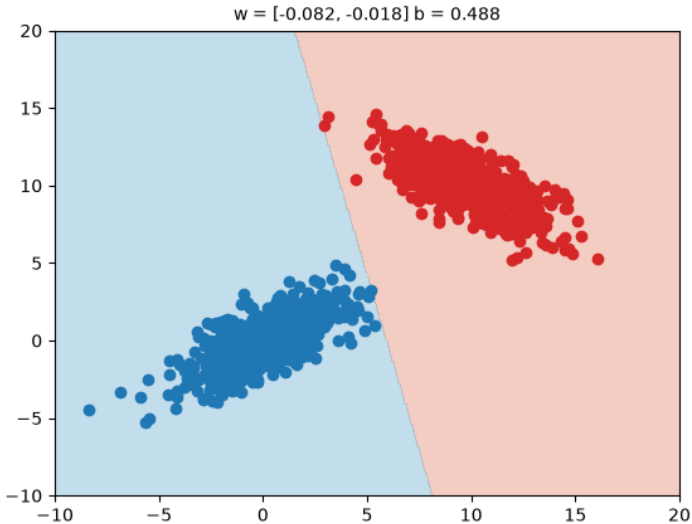
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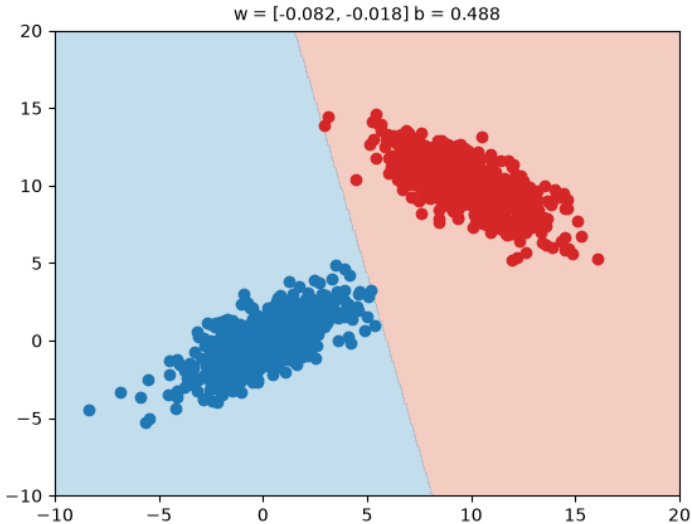
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Perceptron Epochs



Perceptron Epochs



Deriving the perceptron algorithm

- Define a cost function / Définissons une fonction de cout:

$$\mathcal{L}(w, b) = - \sum_{\forall c_n \neq \text{sign}(w^\top x_n + b)} c_n (w^\top x_n + b)$$

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$$w = w - \eta \nabla_w \mathcal{L}(w, b), \qquad b = b - \eta \nabla_b \mathcal{L}(w, b)$$

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- The gradient: / Le gradient:

$$\nabla_w \mathcal{L}(w) = - \sum_{\forall c_n \neq \text{sign}(w^\top x_n + b)} c_n x_n, \qquad \nabla_b \mathcal{L}(b) = - \sum_{\forall c_n \neq \text{sign}(w^\top x_n + b)} c_n$$

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- The updates / Les updates: (Same as the perceptron algo!/ Les memes updates qu'algo perceptron!)

$$w = w + \eta \sum_{\forall c_n \neq \text{sign}(w^\top x_n + b)} c_n x_n, \qquad b = b + \eta \sum_{\forall c_n \neq \text{sign}(w^\top x_n + b)} c_n$$

But can we do something better?

- Perceptron is basically / Perceptron est simplement:

$$f(x) = u(w^\top x + b)$$

where $u(.)$ is a step function (or sign(.) depending on how we define c_n) . / où $u(.)$ est un 'step function'.

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  - À cause de la step function on ne peut pas utiliser la modèle directement dans le fonctionne de cout.
- What if we want to have smooth, differentiable transitions? / Et si on veut avoir des transitions plus smooth?

Table of Contents

Classification Intro

Generative Classification

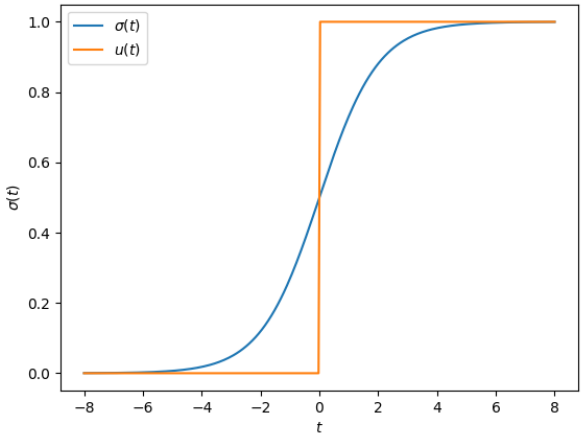
Discriminative Classification
- Linear Classifiers
- The perceptron algorithm
- Logistic Regression

Non-Linear Classification
- Kernel Logistic Regression
- Neural Network Classification

Evaluating Classifiers

The logistic function

- We can use the logistic function $\sigma(t) = \frac{1}{1+e^{-t}}$. / On peut utiliser la fonction logistique.



Where does the sigmoid come from?

- Back to generative classification with two classes. Bayes rule gives / Revenons à la classification générative. La règle de Bayes donne:

$$p(c=1|x) = \frac{p(x|c=1)\,p(c=1)}{p(x|c=1)\,p(c=1) + p(x|c=2)\,p(c=2)} = \frac{1}{1+e^{-a(x)}} = \sigma(a(x))$$

  - with $a(x) = \log \frac{p(x|c=1)\,p(c=1)}{p(x|c=2)\,p(c=2)}$: our discriminant $d(x)$ plus the log ratio of the class priors. / Notre $d(x)$ plus le log ratio des priors.

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- If both classes are Gaussian with the same covariance Σ, the $x^\top \Sigma^{-1} x$ terms cancel and $a(x)$ becomes linear: / Si les deux classes sont gaussiennes avec la même covariance, $a(x)$ devient linéaire:

$$a(x) = w^\top x + b, \quad w = \Sigma^{-1}(\mu_1 - \mu_2), \quad b = -\tfrac{1}{2}\mu_1^\top \Sigma^{-1}\mu_1 + \tfrac{1}{2}\mu_2^\top \Sigma^{-1}\mu_2 + \log \frac{p(c=1)}{p(c=2)}$$

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- So $p(c=1|x) = \sigma(w^\top x + b)$: exactly the logistic function! Logistic regression learns w, b directly, without going through μ_1, μ_2, Σ. (Bishop 4.2)
  - Exactement la fonction logistique! La régression logistique apprend w, b directement.

Logistic regression

- Set the estimator as $f(w, b) = \sigma(w^\top x + b)$.

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- Same neural network, different activation function. / La meme réseau de neurones, different fonctionne d'activation.
- What will be the loss function? / Quelle sera la fonctionne de coût?
- How about the negative Bernoulli log-likelihood?

Bernoulli Distribution

- $\mathcal{BE}(y;\pi) = \pi^y(1-\pi)^{1-y}$, $y \in \{0,1\}$.
- Let's take the log / Prenons le logarithme

$$\log \mathcal{BE}(y;\pi) = y \log \pi + (1-y)\log(1-\pi)$$

.

- We will parametrize the Bernoulli distribution with $\pi = \sigma(w^\top x + b)$/ On est en train de parametriser la distribution Bernoulli.

Logistic regression loss function

- Training loss / La fonction de cout pour l'entrainement:

$$\begin{aligned}\mathcal{L}(w, b) &= \sum_n \left(-y_n \log \pi_w(x_n) - (1 - y_n) \log(1 - \pi_w(x_n))\right) \\ &= \sum_n \left(-y_n \log \left(\sigma(w^\top x_n + b)\right) - (1 - y_n) \log \left(1 - \sigma(w^\top x_n + b)\right)\right)\end{aligned}$$

- Let's calculate the gradient with respect to w. / Calculons le gradient par rapport à w.

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- Few more things: $\frac{d\sigma(x)}{dx} = \sigma(x)(1 - \sigma(x))$, $\frac{d \log x}{dx} = 1/x$.

The gradient wrt w

- Just do it! / Faisons-le!

$$\frac{d\mathcal{L}(w,b)}{dw} = -\sum_n y_n \frac{\sigma(w^\top x_n + b)}{\sigma(w^\top x_n + b)}(1 - \sigma(w^\top x_n + b))x_n$$
$$+ \sum_n (1 - y_n)\frac{1 - \sigma(w^\top x_n + b)}{1 - \sigma(w^\top x_n + b)}\sigma(w^\top x_n + b)x_n$$

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- Makes a lot of sense! / Ça fait du sense!

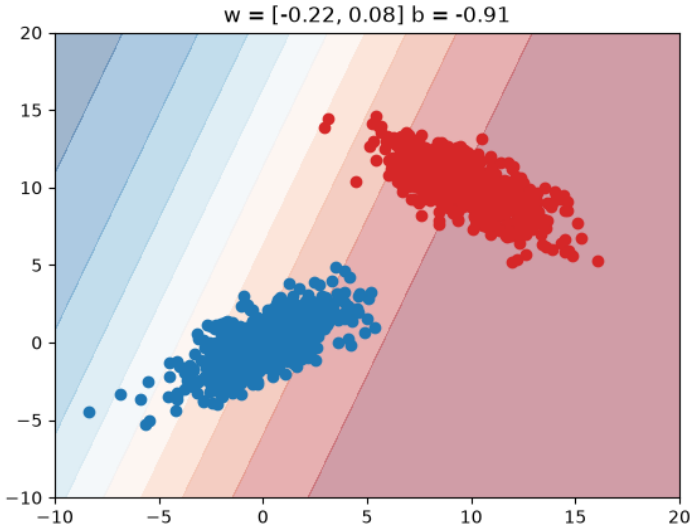
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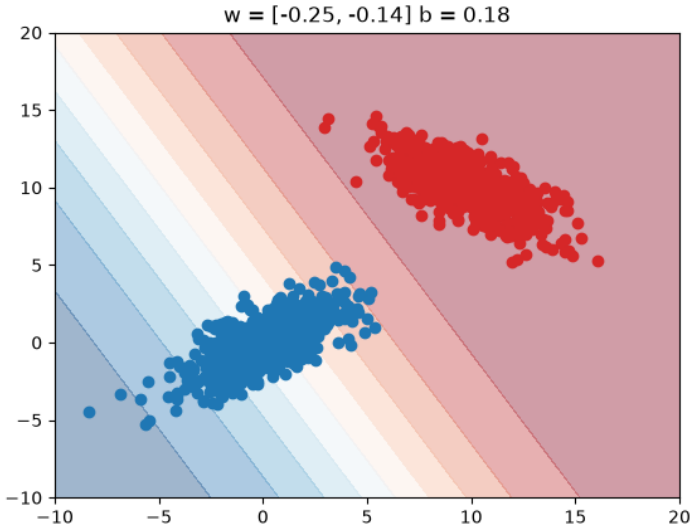
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- Makes a lot of sense! / Ça fait du sense!
- torch will automatically do this for us. However, one needs to do this at least once in their life!
  - torch fait ça automatiquement mais on doit faire cette exercise au moins une fois dans notre vie!

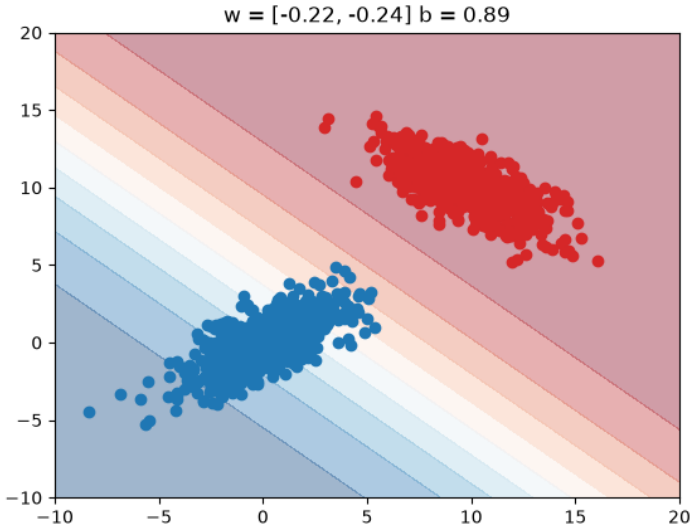
Logistic Regression Epochs



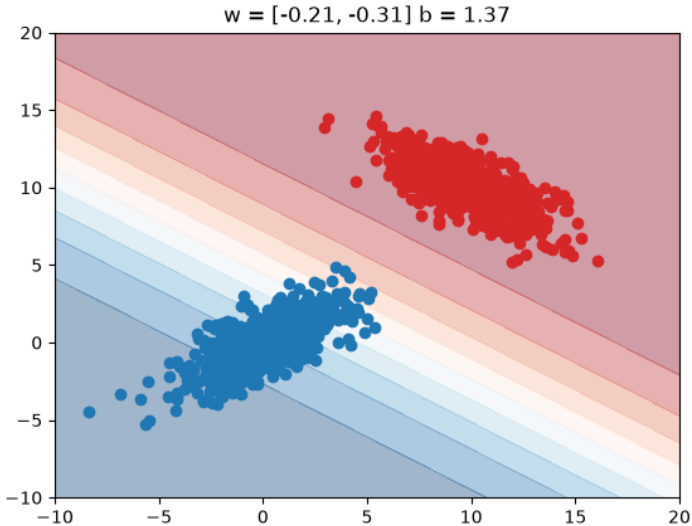
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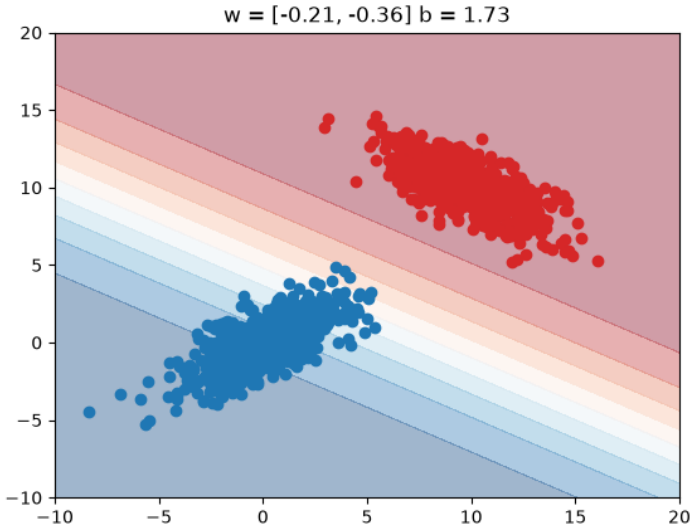
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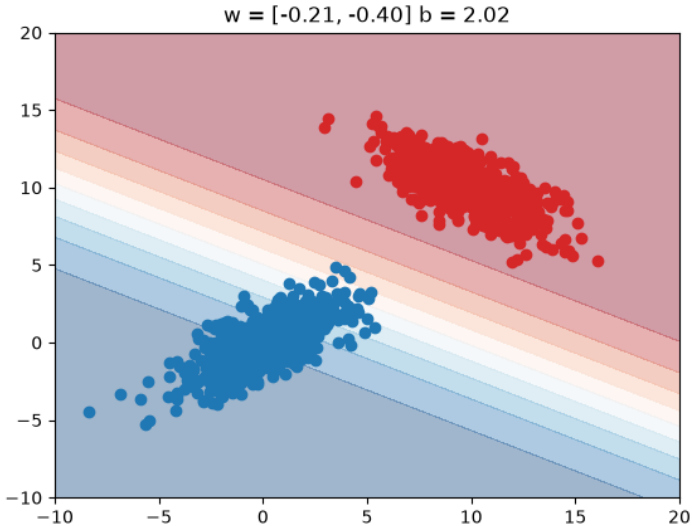
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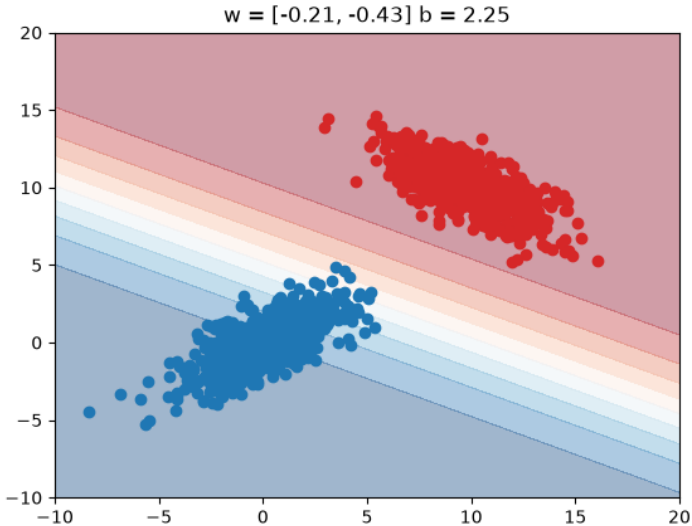
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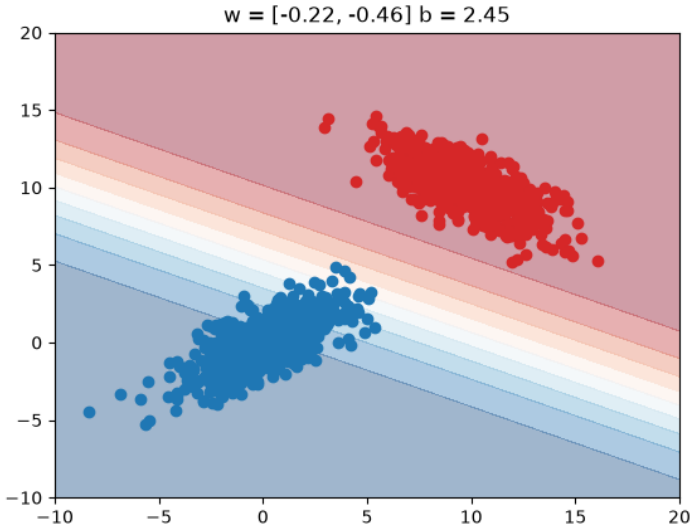
Logistic Regression Epochs



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Logistic Regression Epochs



Generative vs Discriminative

Generative: fit $p(x|c)$ and $p(c)$
Générative: on modélise $p(x|c)$ et $p(c)$

- Can generate samples and flag outliers (low $p(x)$). / Peut générer des échantillons et détecter les outliers.
- Handles missing features. A new class can be added without retraining the others. / Gère les features manquantes. On peut ajouter une classe sans réentraîner les autres.
- Needs fewer examples, if its assumptions are right. / Besoin de moins de données, si les hypothèses sont bonnes.

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Discriminative: fit $p(c|x)$ directly
Discriminative: on modélise $p(c|x)$ directement

- Fewer parameters: $L + 1$ for logistic regression vs. $2L + L(L + 1)/2 + 1$ for two Gaussians with a shared Σ. / Moins de paramètres.
- Fewer assumptions: works even if $p(x|c)$ is not Gaussian. / Moins d'hypothèses.
- Usually more accurate when there is a lot of data. / Souvent plus précis avec beaucoup de données.

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- Ng & Jordan (2002), naive Bayes vs logistic regression: the generative model gets close to its best error with fewer examples, the discriminative one ends with a lower error given enough data. / Le modèle génératif converge plus vite, le discriminatif finit avec une erreur plus basse.

Generalizing to multiway classification

- Define (a new network!) / On définie une nouvelle réseau

$$\pi_k := \exp(w_k^\top x + b_k) / \sum_l \exp(w_l^\top x + b_l)$$

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&= -\sum_n \sum_k [y_n = k] \log \pi_k
\end{aligned}$$

Generalizing to multiway classification

- Define (a new network!) / On définie une nouvelle réseau

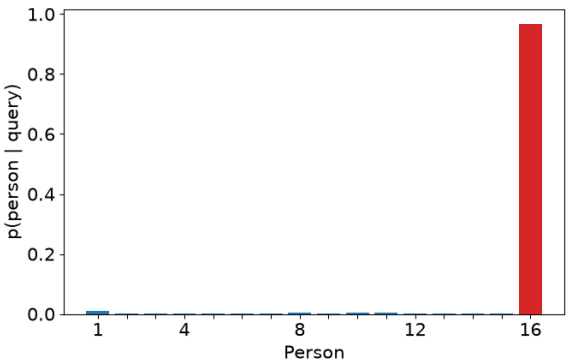
$$\pi_k := \exp(w_k^\top x + b_k) / \sum_l \exp(w_l^\top x + b_l)$$

- The loss function is now going to be the log-likelihood of discrete distribution / La fonctionne de cout sera maintenant le log-likelihood de le distribution discrète.

$$\begin{aligned}
\mathcal{L}(W, b) &= -\log\left(\prod_n \text{Discrete}(y_n; \pi_{1:K})\right) \\
&= -\log\left(\prod_n \prod_k \pi_k^{[k=y_n]}\right) \\
&= -\sum_n \sum_k [y_n = k] \log \pi_k \\
&= -\sum_n \sum_k [y_n = k] \left(w_k^\top x_n + b_k - \log \sum_l \exp(w_l^\top x_n + b_l)\right)
\end{aligned}$$

Back to our faces

- Let's apply this multiway logistic regression to our faces: raw pixels, trained on 7 images per person, tested on the other 3 (48 test faces, including our query). / Appliquons-la à nos visages: entraînée sur 7 images par personne, testée sur les 3 autres.

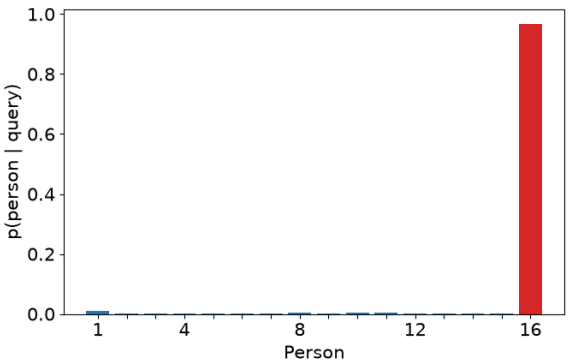




Inner products said: person 1 (wrong)
Softmax says: person 16 (right, $p = 0.97$)

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Method	Test acc.
Inner product, 1 template	69%
Inner product, nearest of 7	92%
Softmax regression	98%

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Generative Classification

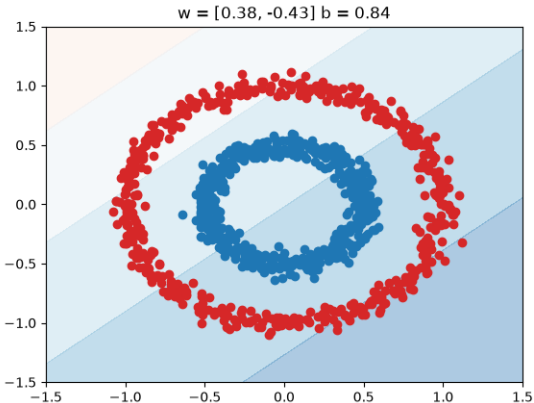
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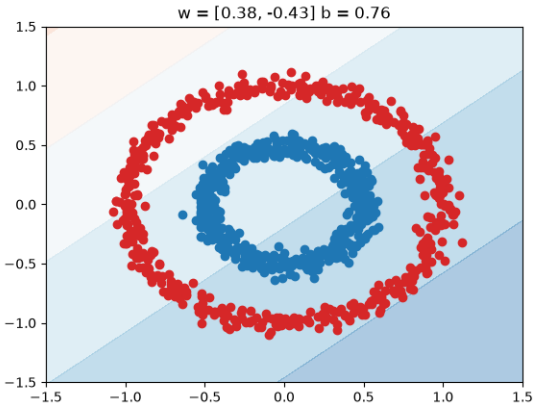
Non-Linear Classification

- What if we have something like this? / Qu'est-ce qu'on fait si on a ça?



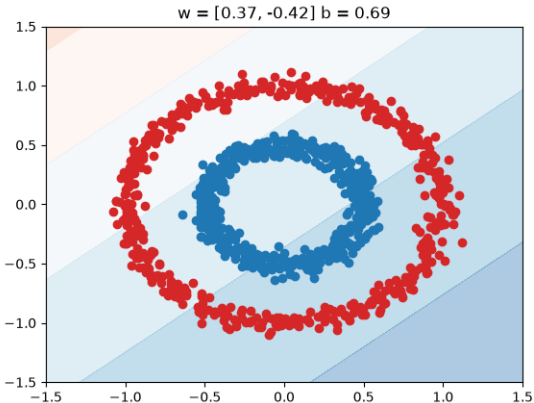
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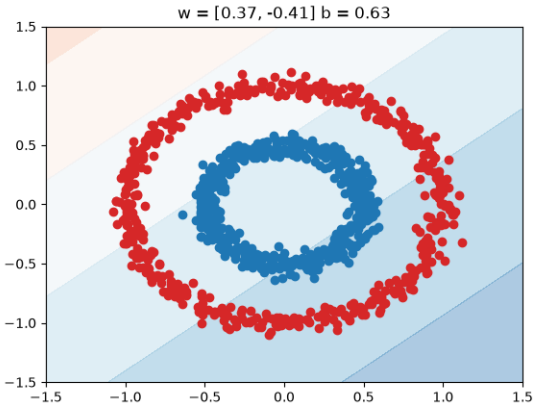
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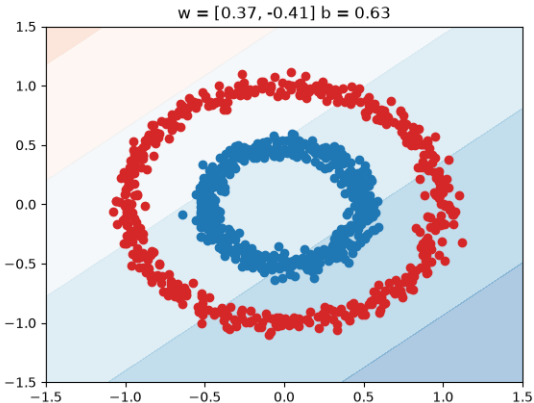
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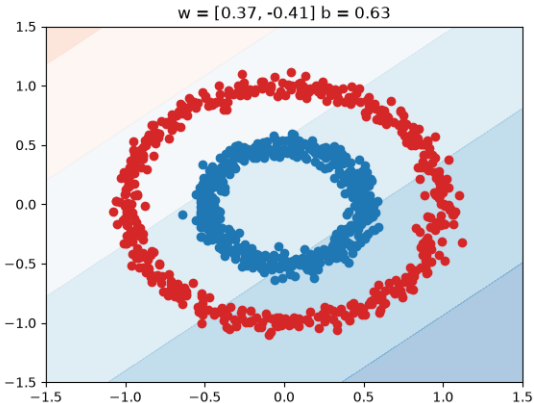
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Non-Linear Classification

- What if we have something like this? / Qu'est-ce qu'on fait si on a ça?



- Linear classifier is not enough! / Classificateur linéaire n'est pas suffisante!
- What can we do? / Qu'est-ce qu'on peut faire?

The Kernel Trick

- We can extend our linear classifier $y = w^\top x \rightarrow y = w^\top \phi(x)$. We introduce a feature transformation $\phi(.)$.
  - On améloire notre classificateur linéaire en introduisant une transformation de feature $\phi(.)$.

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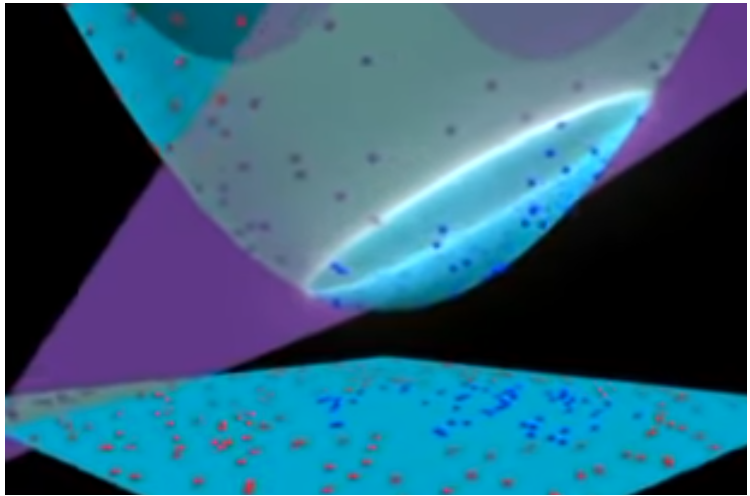
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- We can incorporate this in many places. But let's look at injecting this idea in logistic regression.
  - On peut incorporer cette idée dans différents place. Mais d'abord essayons d'injecter cette idée dans la regression logistique.

Motivating the Kernel Trick



Let's watch!

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Kernel Logistic Regression

- The coin bias is now estimated with the kernel-trick / Le biais est maintenant estimé avec le kernel trick.

$$\pi = \sigma\left(\sum_n a_n k(x, x_n)\right)$$

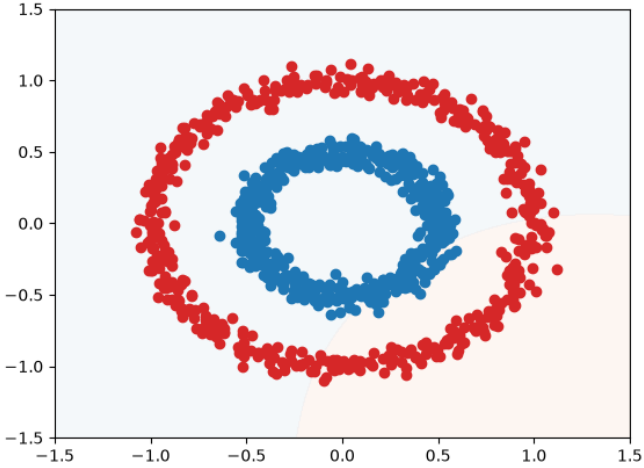
- Then the loss becomes, / le fonctionne de cout devient,

$$\begin{aligned}\mathcal{L}(a) = \sum_n \Big(& - y_n \log\Big(\sigma\Big(\sum_j a_j k(x_n, x_j)\Big)\Big) \\ & - (1 - y_n) \log\Big(1 - \sigma\Big(\sum_j a_j k(x_n, x_j)\Big)\Big)\Big)\end{aligned}$$

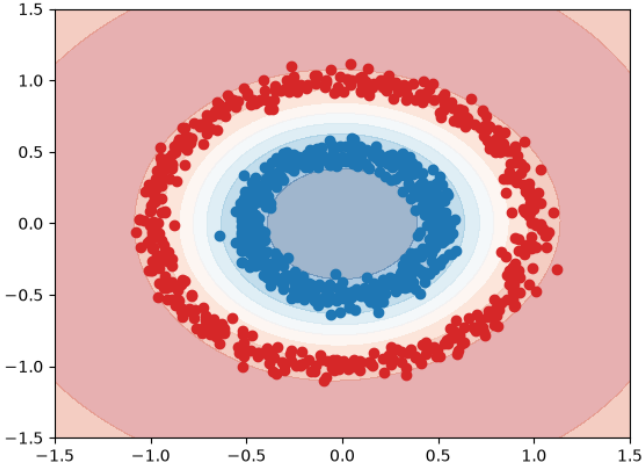
- Kernel functions are chosen from options such as RBF Kernel, Polynomial Kernel,.. / On choisit le kernel entre les options qu'on est habitué.

$$k_{\text{rbf}}(x, x_n) = \exp(-\gamma \|x - x_n\|_2^2)$$

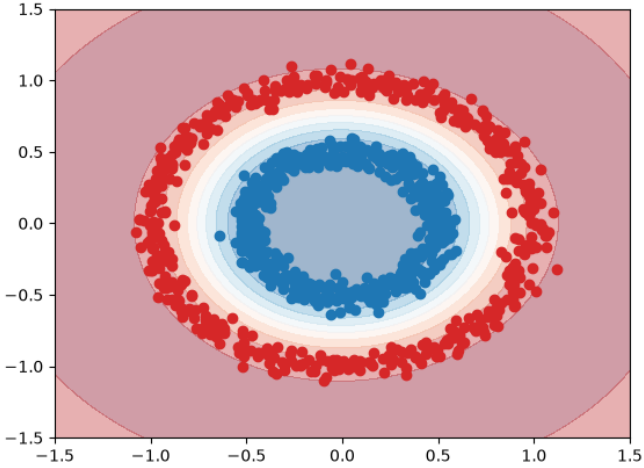
Kernel Logistic Regression Learning Steps



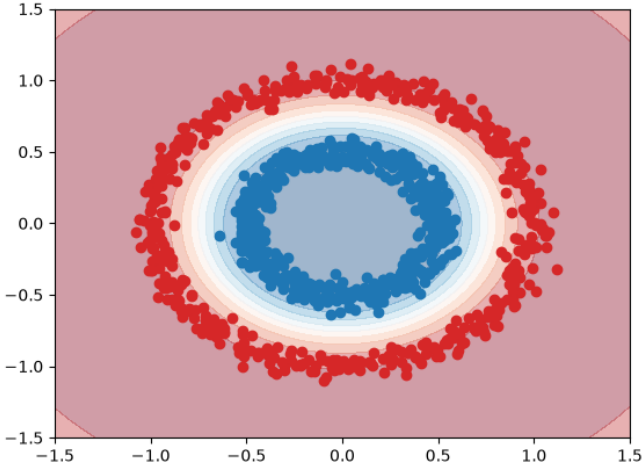
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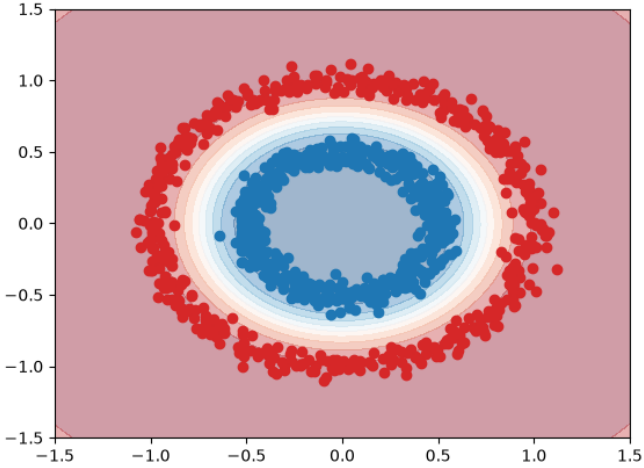
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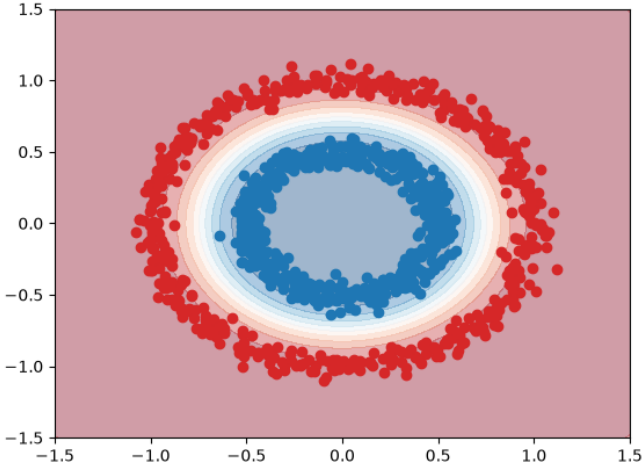
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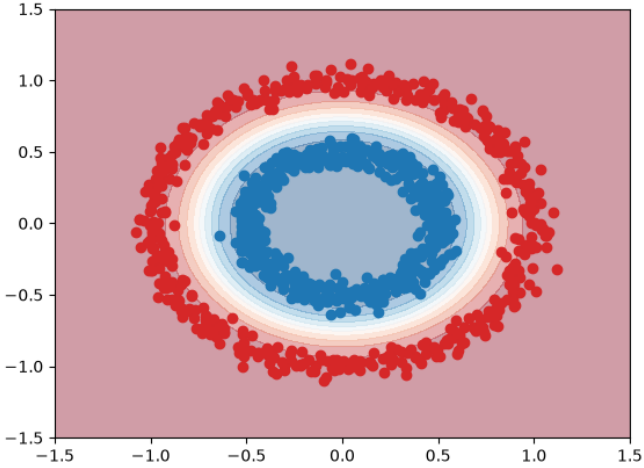
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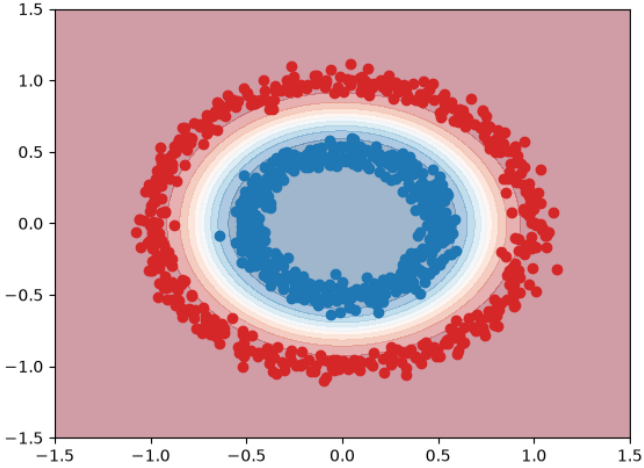
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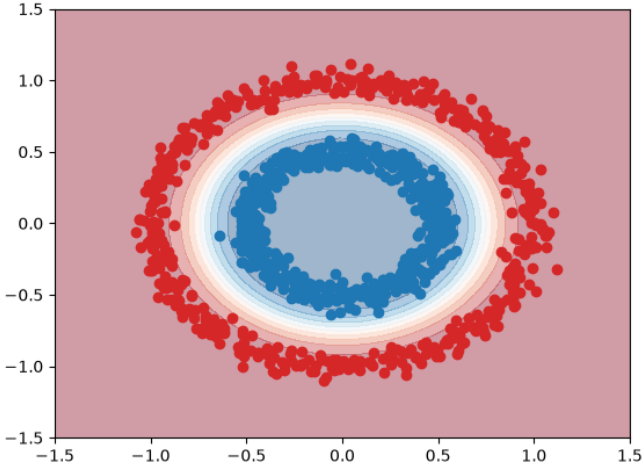
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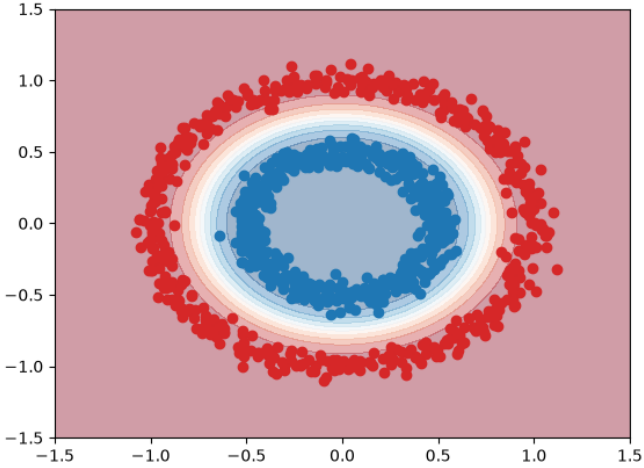


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Neural Network Classification

- The bias parameter can be estimated with a general function $f(x)$.

$$\begin{aligned}\pi =& f(x)\\ =& \sigma(w_M^\top h(W_{M-1}^\top h(W_{M-2}^\top(\dots h(W_1^\top x)))))\end{aligned}$$

- The general function maps x from L dimensions to K_1, K_2, $\dots$, K_{M-1} dimensions. The last layer w_M is a vector and maps to 1 output dimension. / Chaque couche map à une dimensionnalité différent K_l.

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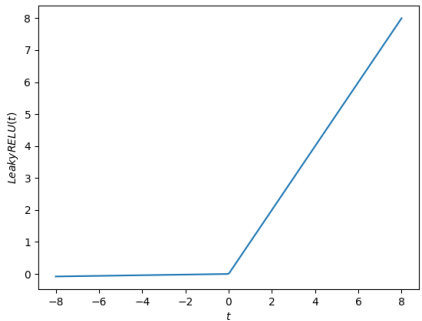
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$$\mathcal{L}(\theta) = \sum_n \Big(- y_n \log\Big(\sigma\Big(f_\theta(x_n)\Big)\Big) - (1 - y_n) \log\Big(1 - \sigma\Big(f_\theta(x_n)\Big)\Big)\Big)$$

An Example Neural Network

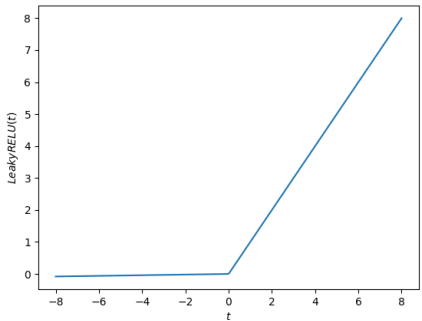
- $f_\theta(x) = w_2 h(W_1^\top x)$, $h(t) = \text{LeakyRELU}(t)$, $W_1 \in \mathbb{R}^{2\times 100}$, $w_2 \in \mathbb{R}^{100}$.



- For the hawk eyed people: We are effectively learning the famous $\phi(.)$ here. Why? / On est en train d'apprendre le fameux $\phi(.)$. Vous voyez pourquoi?

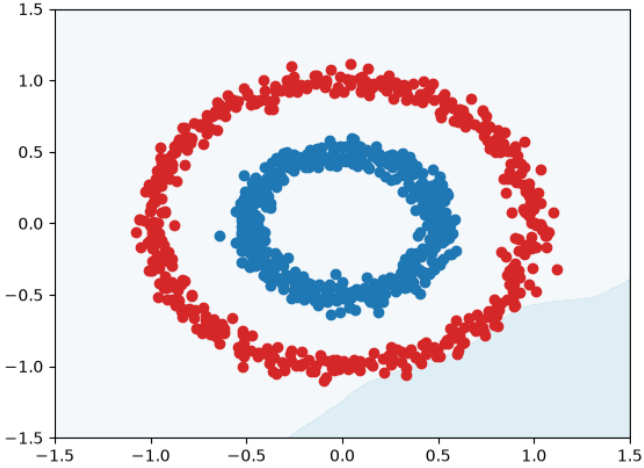
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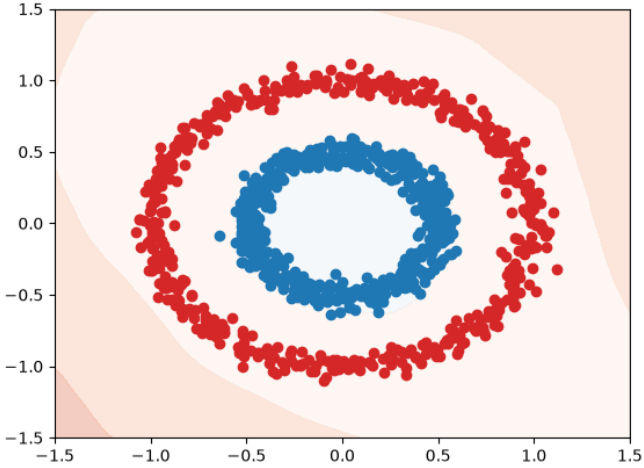


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- Let's say we define $\phi_{W_1}(x) := h(W_1^\top x)$.

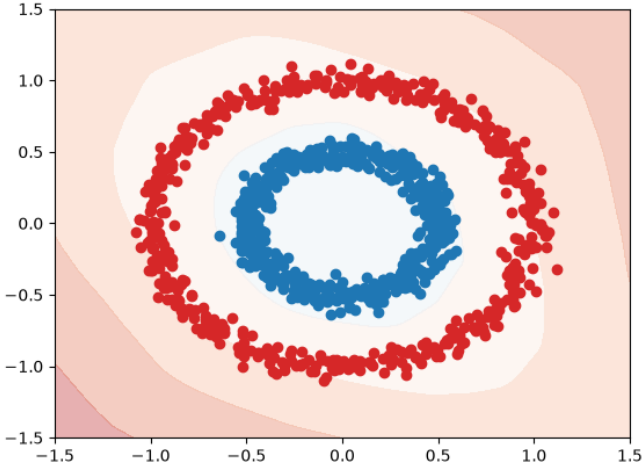
Neural Network Classifier Learning Steps



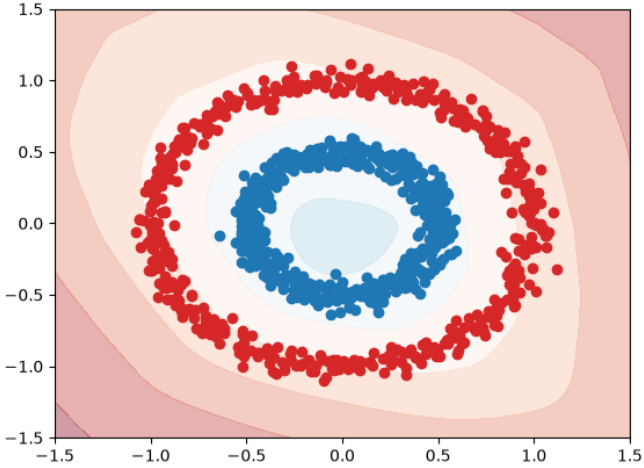
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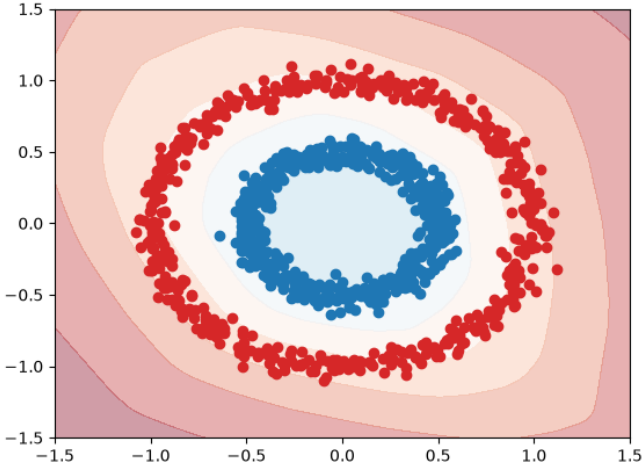
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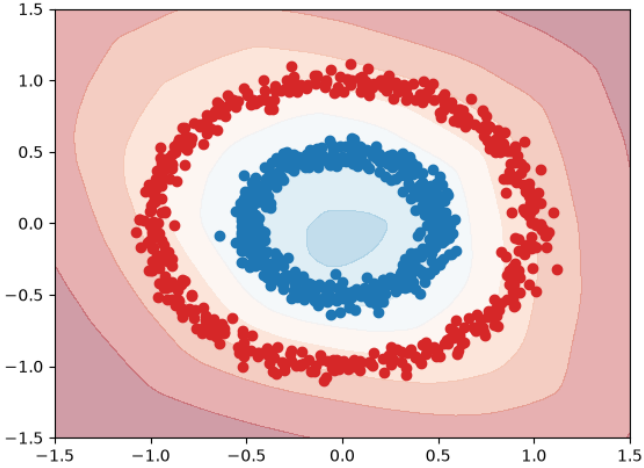
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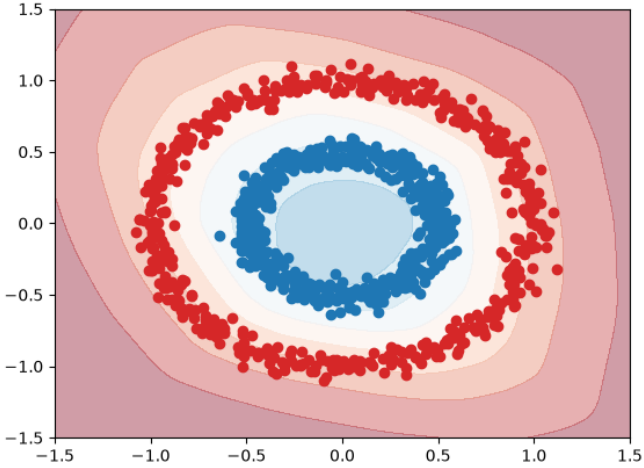
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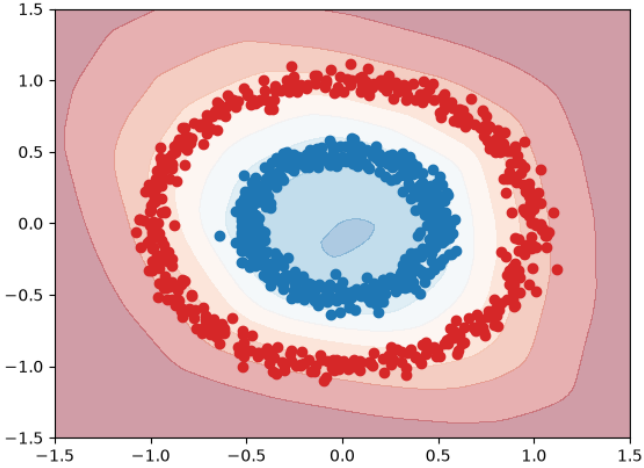
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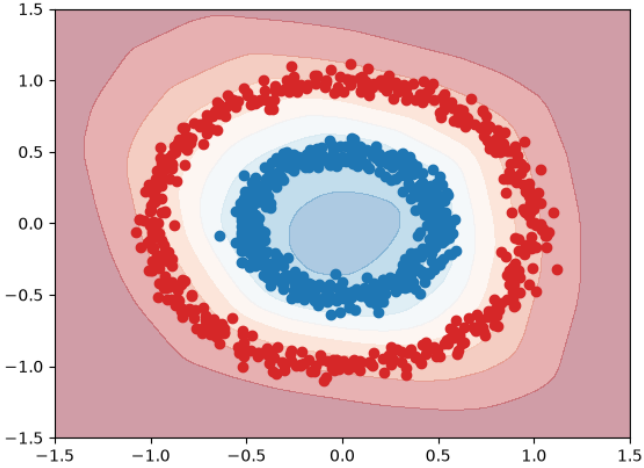
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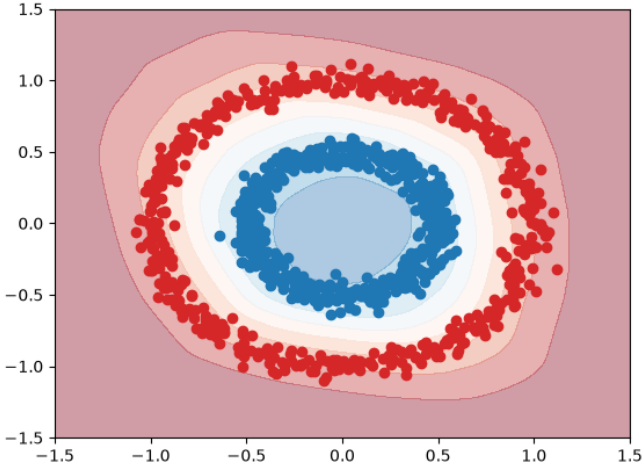
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Neural Network Classifier Learning Steps



Kernel Methods vs Neural Networks

- Kernel Methods are not very suitable for large datasets because of the hard dependency on the training set $x_1, \ldots, x_N$. / Les méthodes de noyau ne sont pas très adaptes pour des datasets larges parce qu'ils dépendent sur le training set.
- Kernel Methods come with an interpretability advantage. At the end of the day it's a linear model, and we have the importance weights. / Les méthodes de noyaux sont plus interpretables car ils sont linéers.
- Neural Nets are more modern because of the way we train them. They more amenable to train with SGD. / Les résaux de nérones sont plus adaptes à entrainer avec SGD sur des grands jeux de données.

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How good is our classifier?

- Always measure performance on data that was not used for training (train / validation / test split). / Toujours évaluer sur des données qui n'ont pas servi à l'entraînement.
  - A flexible model (kernels, neural nets) can fit the training set almost perfectly and still fail on new data. / Un modèle flexible peut fitter le training set presque parfaitement et échouer sur des nouvelles données.

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	predicted positive	predicted negative
actually positive	true positive (TP)	false negative (FN), miss
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- accuracy $= \frac{TP+TN}{N}$, precision $= \frac{TP}{TP+FP}$, recall $= \frac{TP}{TP+FN}$
  - Precision: when we say "detected", how often are we right? Recall: what fraction of the true events do we find?
  - Précision: quand on détecte, a-t-on raison? Rappel: quelle fraction des vrais événements trouve-t-on?

Accuracy can be misleading

- Detect a rare sound event (e.g. glass breaking) in 1000 audio frames. Only 10 frames contain it. / Détecter un événement sonore rare dans 1000 trames audio. Seulement 10 trames le contiennent.

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Always say "no event"

	pred. +	pred. −
actually +	0	10
actually −	0	990

accuracy = 99%

recall = 0%

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recall = 0%

A real detector

	pred. +	pred. −
actually +	8	2
actually −	20	970

accuracy = 97.8%
precision = 29%, recall = 80%

Accuracy can be misleading

- Detect a rare sound event (e.g. glass breaking) in 1000 audio frames. Only 10 frames contain it. / Détecter un événement sonore rare dans 1000 trames audio. Seulement 10 trames le contiennent.

Always say "no event"

	pred. +	pred. −
actually +	0	10
actually −	0	990

accuracy = 99%
recall = 0%

A real detector

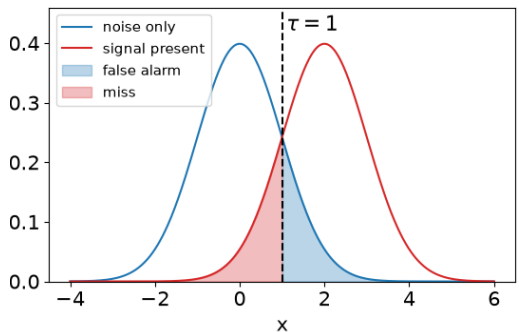
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actually +	8	2
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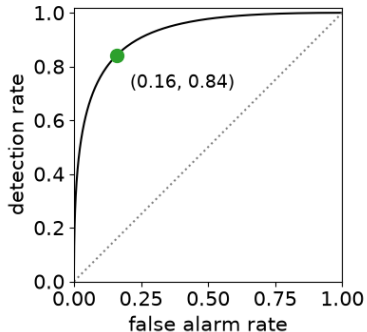
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- Lower accuracy, but only the second one is useful! With imbalanced classes, look at precision and recall. / Accuracy plus basse, mais seul le deuxième est utile! Avec des classes déséquilibrées, regardez la précision et le rappel.

Detection: choosing the threshold

- Same two Gaussians as before: noise only $x \sim \mathcal{N}(0,1)$, signal present $x \sim \mathcal{N}(2,1)$. We decide "signal" if $x > \tau$. / Même exemple qu'avant. On décide "signal" si $x > \tau$.

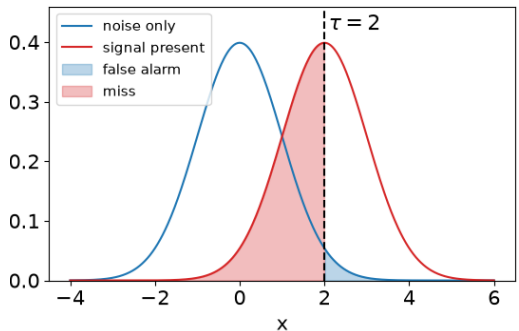


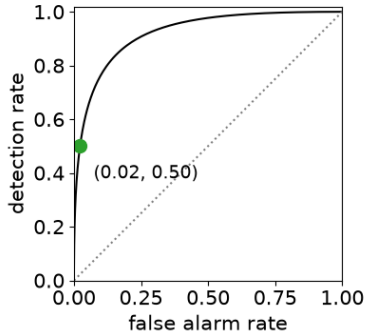


- $\tau = 1$ is the $d(x) = 0$ boundary we derived before. / C'est la frontière $d(x) = 0$ qu'on a dérivée avant.

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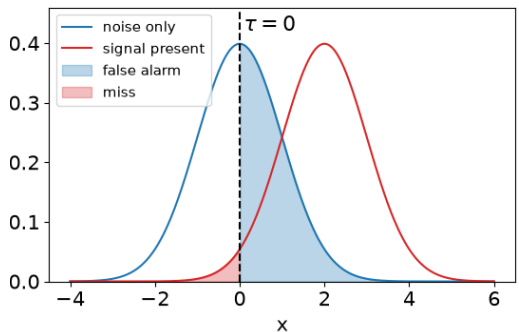


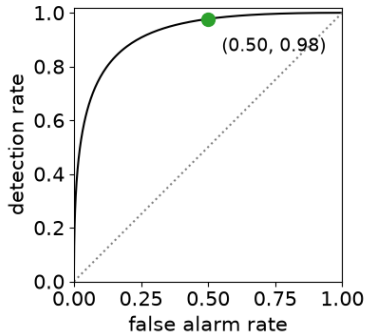


- Raising τ: fewer false alarms, but more misses. / Moins de fausses alarmes, mais plus de détections manquées.

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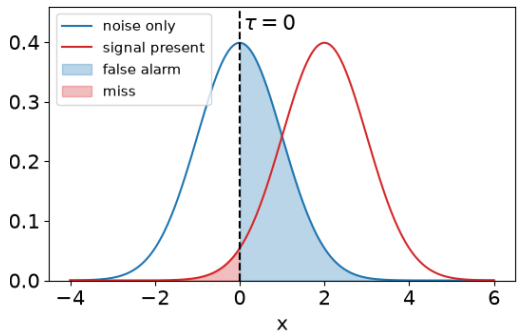


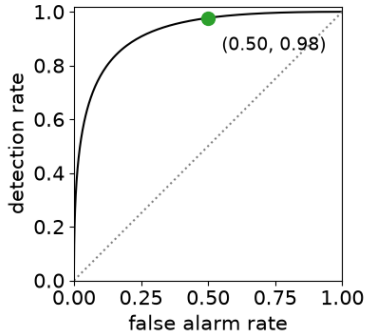


- Lowering τ: fewer misses, but more false alarms. / Moins de détections manquées, mais plus de fausses alarmes.

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- Sweeping τ traces the ROC curve (right). Area under it (AUC): 1 is perfect, 0.5 is chance (dotted). Here AUC $= 0.92$. / En variant τ on obtient la courbe ROC.

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  - Discriminative Classification

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- We have not talked about SVMs. It's an important idea to know, but you can get that from many other classes. On n'a pas parlé de SVMs.

Suggested reading

- Bishop, chapters 4, 5, 6

Next week

- More deep learning!