IFT 4030/7030, Machine Learning for Signal Processing
Week6: Machine Learning 3, Classification

Cem Subakan

Admin

- How is homework 1 going? The deadline is Oct 24th (as indicated on teams)
  - Comment va le devoir 1? Le deadline est le Oct 24. (comme indiqué sur teams)
- If you have questions on your project just let me know.
  - S'il y a des hesitations laissez nous savoir.
- Did you manage to get into VALERIA? There will be a tutorial.
  - Ca va bien avec VALERIA? Il y aura un tutoriel.
- Aujourd'hui: Classification

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Supervised Learning

- So far we have mostly done unsupervised learning to discover structures.
  - Jusqu'à maintenant on a majoritairement fait de l'apprentissage non-supervisé. Le but était de découvrir la structure.
- Now, we will do classification / detection, or supervised learning in other words.
  - Maintenant on va faire de l'apprentissage supervisé.

A simple example

Query

Classes

A simple example

Query

Classes

How can we assign the query to a class? / Comment peut-on assigner l'exemple en question à une classe?

We can simply calculate inner products!

We can calculate inner products! / On peut calculer des produits scalaires!

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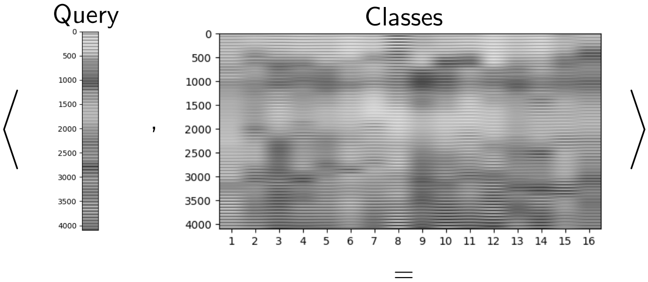

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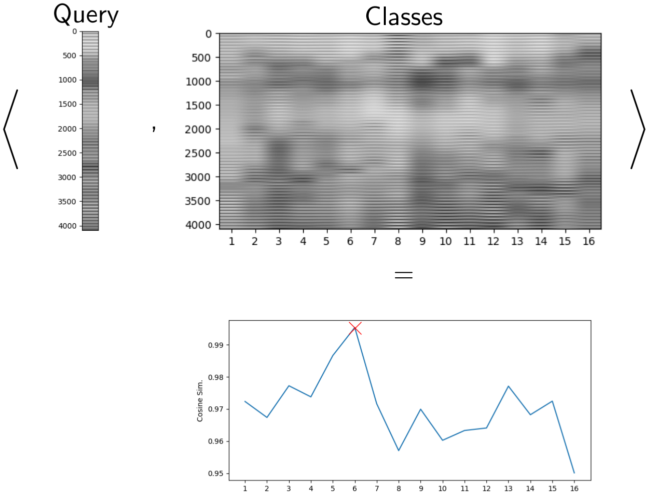

It worked but will it this time?

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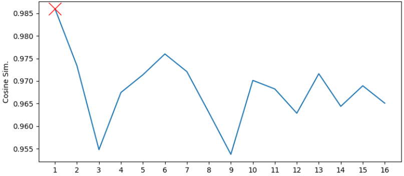

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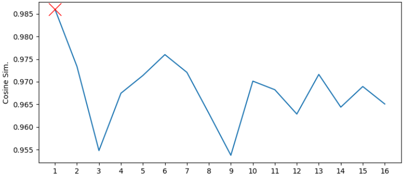

Didn't work this time!/Ça pas fonctionné!

What can we do to fix this?

- We will take a statistical approach (as usual). This will consider the class distributions. / On va prendre une approche de statistique comme d'habitude qui va prendre en compte les distributions de classes.

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- Approach 1: Generative Classification
  - Approche 1: Classification Générative
    - We will learn a distribution over samples in the class. / On va fitter une distribution sur les échantillons dans la classe.
- Approach 2: Discriminative Classification
  - Approche 2: Classification Discriminative
    - We will learn how the class distribution separate. / On va fitter une fonction pour comprendre comment les classes se séparent.

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- We will build step by step towards neural nets, and why we need them in this lecture. / On va builder graduellement à pourquoi on a besoin des réseaux de neurones dans ce cours.
  - Linear Classifiers (Perceptron Algo. Logistic Regression, Kernel Methods, and then Multilayer perceptron)

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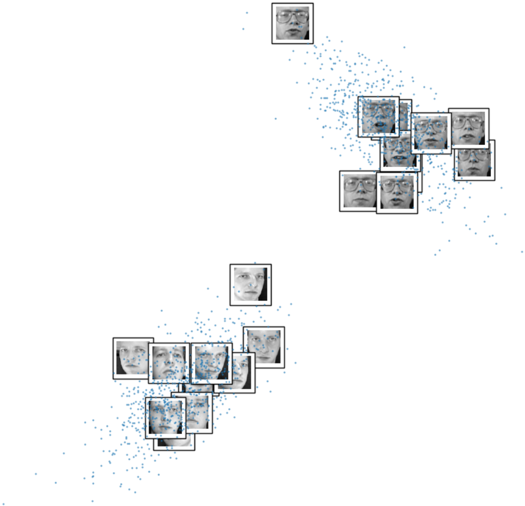
Generative Classification

- Generative Classification fits distributions to each class / Classification générative fit une distribution à chaque classe.



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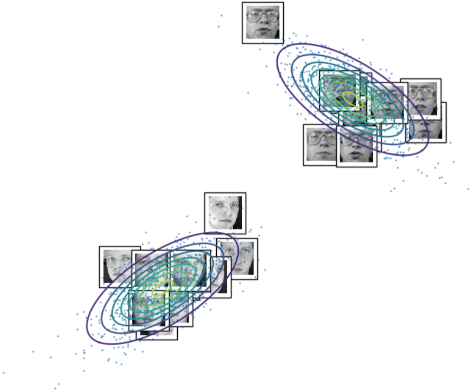

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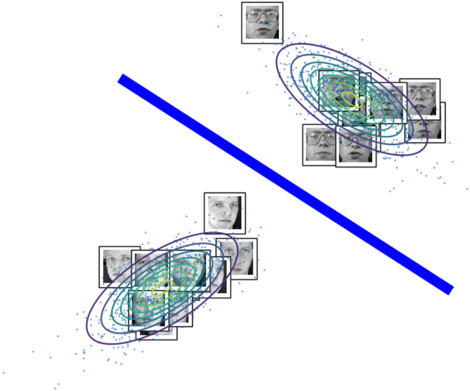


Generative Classification

- Training Time: Fit a distribution $p(x|\theta_k)$ to each class k with maximum likelihood.
  - L'entrainement: On va fitter une distribution $p(x|\theta_k)$ pour chaque classe k avec maximum likelihood.

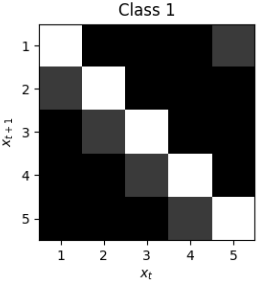
$$\max_{\theta_k} \sum_{n \in \text{class } k} \log p(x_n|\theta_k)$$

- Test Time: Evaluate the likelihood for each model. Assign to the largest likelihood class!
  - L'entrainement: Evaluez le likelihood pour chaque modèle. Assignez à la classe avec le likelihood plus grand.

$$\widehat{c} = \arg\max_k \log p(x_{\text{test}}|\theta_k)$$

Example Application: Classifying Sequences

- class 1 $\sim$ Markov(A_1)
- class 2 $\sim$ Markov(A_2)



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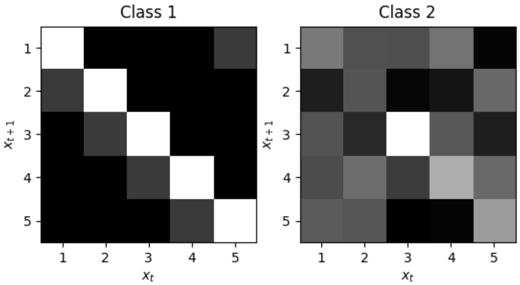

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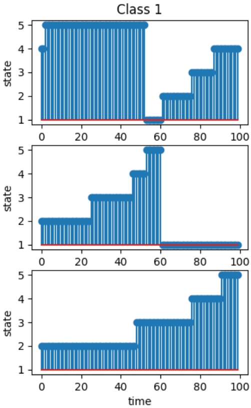

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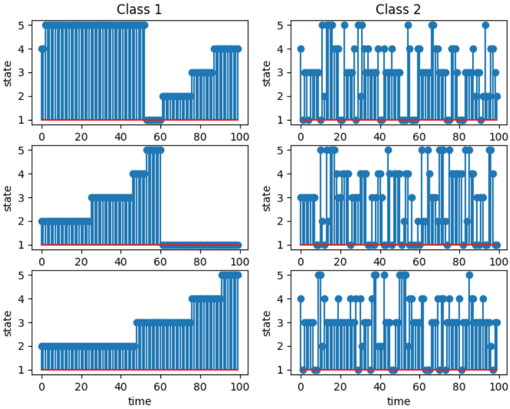
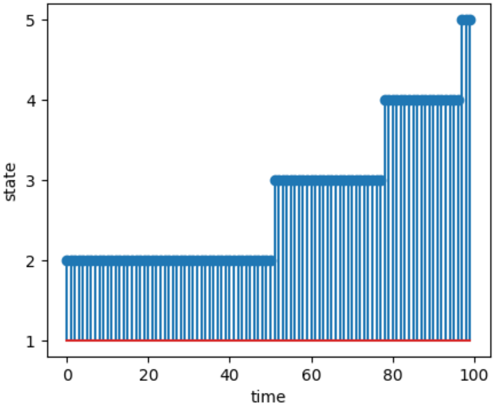


Test time

- Here's a test sequence / Une séquence de test



- Do you think it belongs to which class? / Vous pensez que cette séquence appertient à quelle classe?

Test time

$$\log p(x_{1:T} | A_k, \pi_k) = \log \left(p(x_1 | \pi_k) \prod_{t=2}^{T} p(x_t | x_{t-1}, A_k) \right)$$

Test time

$$\log p(x_{1:T}|A_k, \pi_k) = \log \left(p(x_1|\pi_k) \prod_{t=2}^{T} p(x_t|x_{t-1}, A_k) \right)$$
$$= \log p(x_1|\pi_k) + \sum_{t=2}^{T} \log p(x_t|x_{t-1}, A_k)$$

Test time

$$
\begin{aligned}
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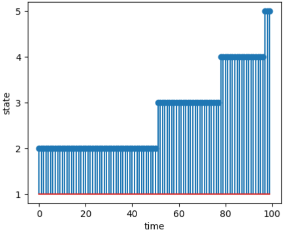
&= \log p(x_1|\pi_k) + \sum_{t=2}^{T} \log p(x_t|x_{t-1}, A_k) \\
&= \log \pi_k + \sum_{t=2}^{T} \log A_k(x_t, x_{t-1})
\end{aligned}
$$

Now, let's calculate

- Observation sequence / Séquence observé



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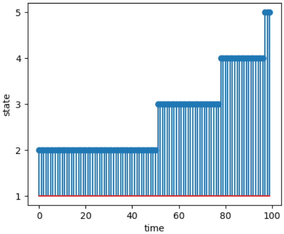
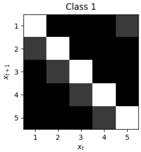
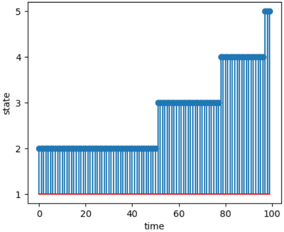


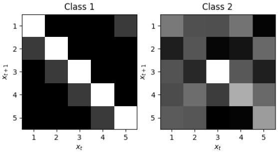


$$
\begin{aligned}
\log p(x_{1:T}|A_1,\pi_1) &= \log\left(\pi_1 \prod_{t=2}^{T} A_1(x_t, x_{t-1})\right) \\
&= \log\left(\frac{1}{5}\cdot 0.95 \cdot 0.95 \cdot 0.95 \ldots\right) \\
&= -15.52
\end{aligned}
$$

Now, let's calculate

- Observation sequence / Séquence observé





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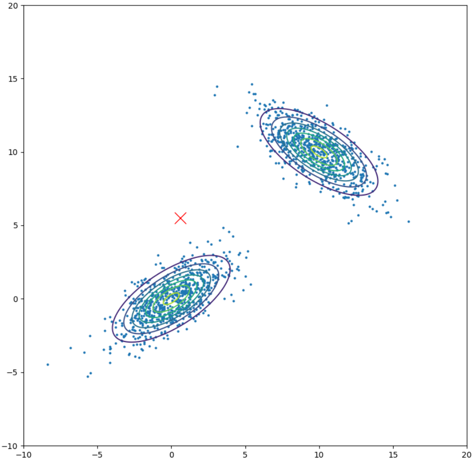
$$= -15.52$$

$$\log p(x_{1:T}|A_2,\pi_2) = \log\left(\pi_1 \prod_{t=2}^{T} A_2(x_t, x_{t-1})\right) = \log\left(\frac{1}{5}\cdot 0.21\cdot 0.21\cdot 0.21\dots\right)$$
$$= -117.93$$

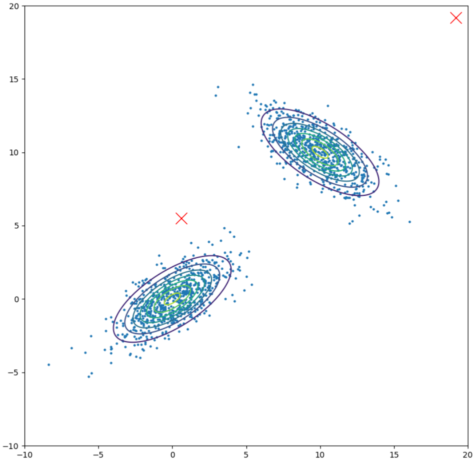
Let's do something simpler

Which class should we assign the red point? / Quel classes doit-on assigner au point rouge?



How about this?

How about this second point? / Et le deuxième point?



The principled way to derive the decision boundaries

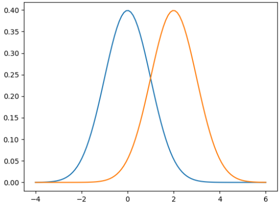
- The discriminant function / La fonctionne de discrimination

$$d(x) = \log \frac{p(x|c=1)}{p(x|c=2)} = \log \frac{p(x|\theta_1)}{p(x|\theta_2)}$$

- The decision boundary: $d(x) = 0$.
- For Gaussians / simple densities this can be derived analytically.

Deriving the decision boundary

- Derive the boundary in 1d case / Dérivons dans le cas 1d.



$$d(x) = \log \frac{p(x|\theta_1)}{p(x|\theta_2)} = \log \frac{\mathcal{N}(0;1)}{\mathcal{N}(2;1)}$$

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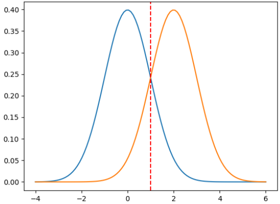
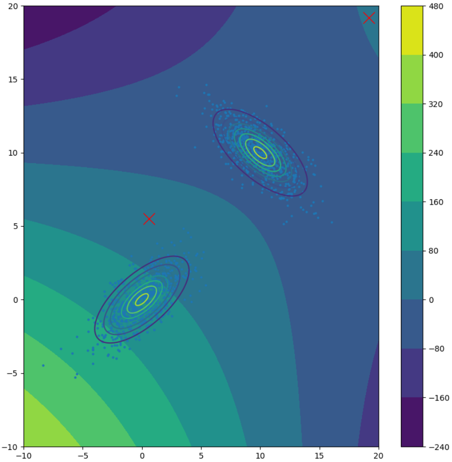


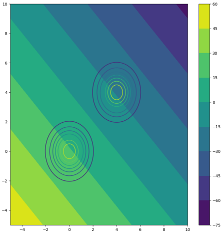
$$
\begin{aligned}
d(x) =& \log \frac{p(x|\theta_1)}{p(x|\theta_2)} = \log \frac{\mathcal{N}(0;1)}{\mathcal{N}(2;1)} \\
=& \log \frac{x^2}{(x-2)^2} = 0 \\
\rightarrow x^2 =& (x-2)^2 \rightarrow x = 1
\end{aligned}
$$

The 2d decision boundary

$$\text{Plot } d(x) = \log \frac{\mathcal{N}([x,y]^\top;\mu_1,\Sigma_1)}{\mathcal{N}([x,y]^\top;\mu_2,\Sigma_2)}$$



Few other cases



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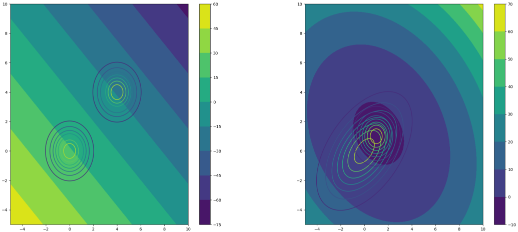

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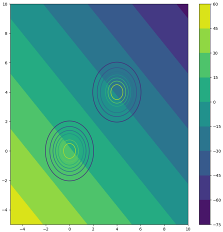

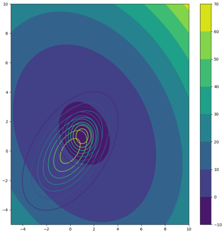

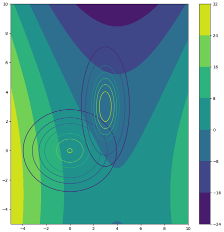

All these cases can be derived analytically! / C'est possible analyticquement calculer

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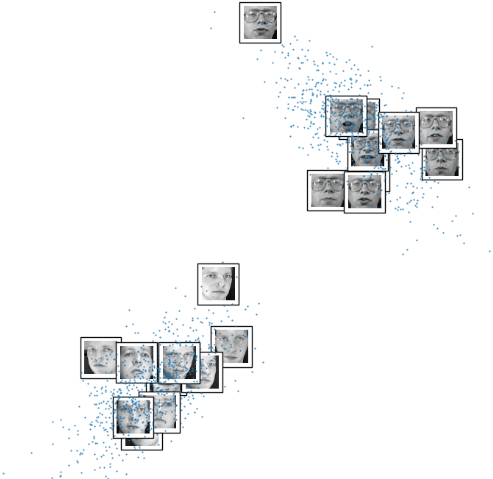
- Discriminative classification direcly learns a decision boundary / Classification discriminative apprend directement un borne de décision.



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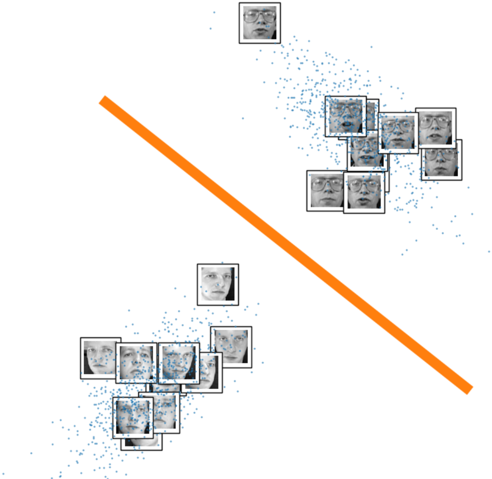

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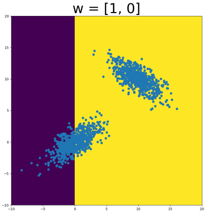

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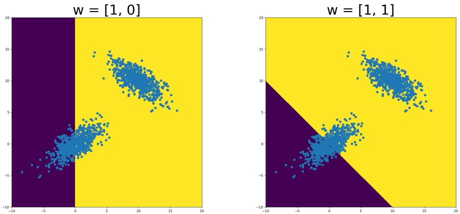

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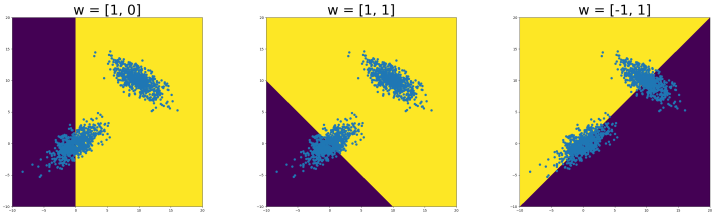

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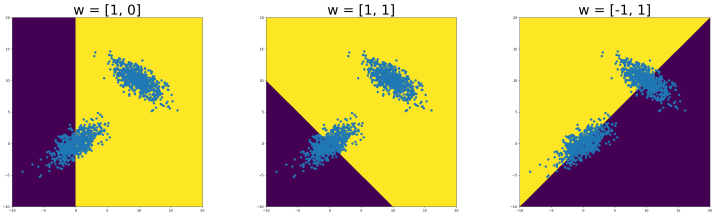

- Btw, we will also add a bias term so that $f(x) = w^\top x + b$ / En passant on va aussi ajouter un biais.

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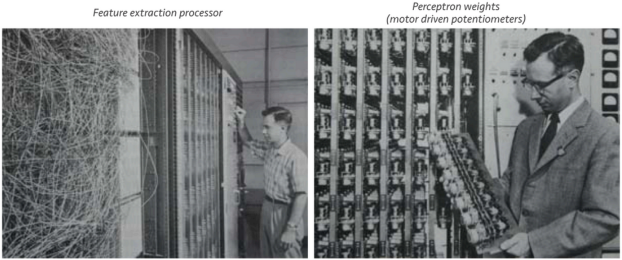
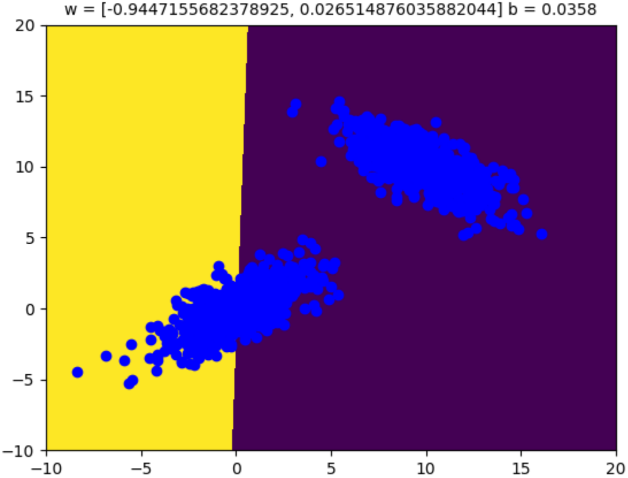
  - Do these updates until convergence. / On répète jusqu'qu'on converge.
  - Note that $c_n \in \{-1, 1\}$. / Notez que $c_n \in \{-1, 1\}$.

The perceptron



Taken from UIUC MLSP class

Perceptron Epochs



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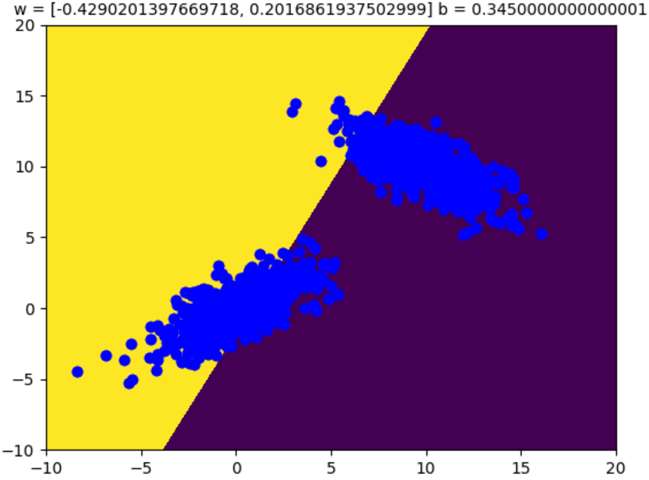

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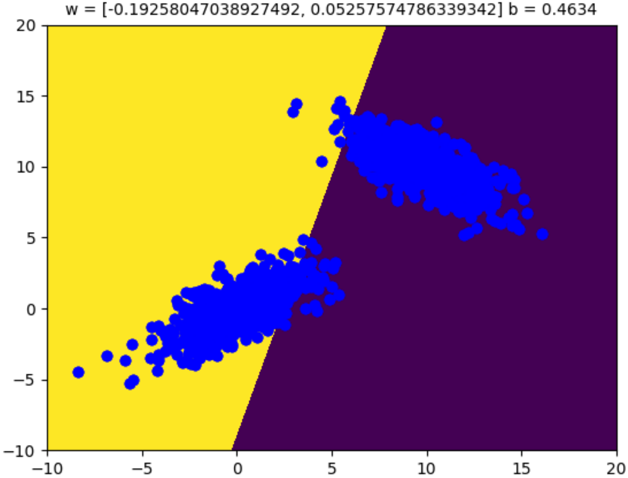

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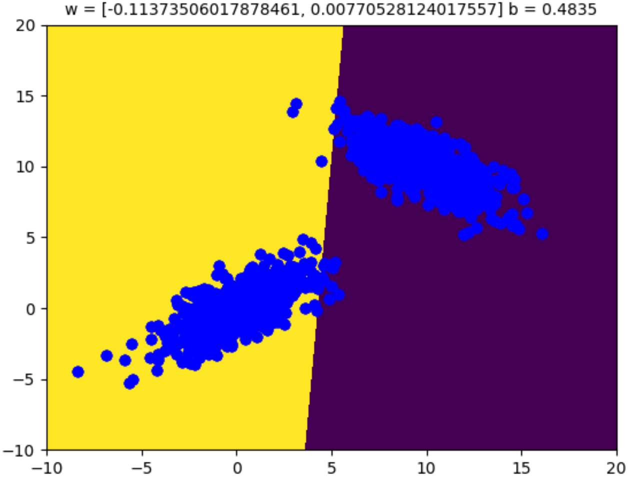

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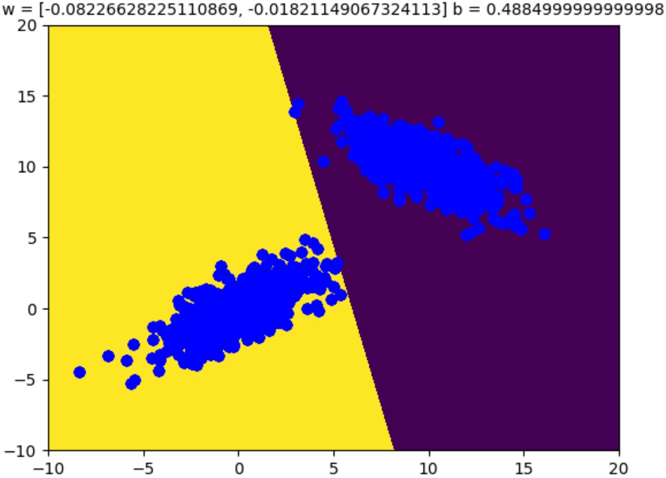

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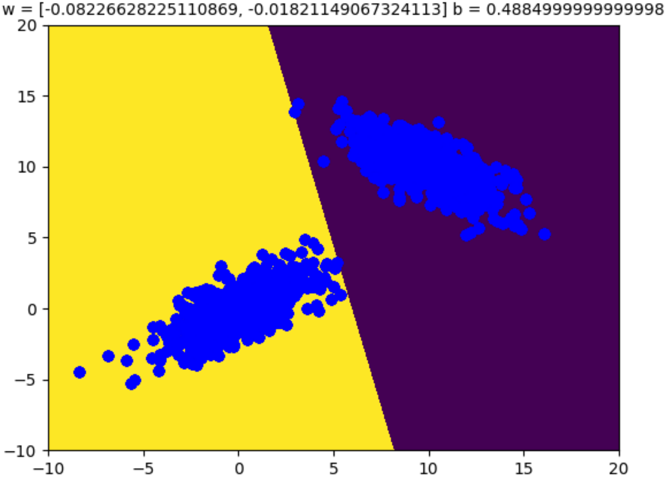

Deriving the perceptron algorithm

- Define a cost function / Définissons une fonction de cout:

$$\mathcal{L}(w, b) = - \sum_{\forall c_n \neq \mathrm{sign}(w^\top x_n + b)} c_n (w^\top x_n + b)$$

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$$\mathcal{L}(w, b) = -\sum_{\forall c_n \neq \text{sign}(w^\top x_n + b)} c_n(w^\top x_n + b)$$

- Do the gradient descent updates: / Faisons les GD updates:

$$w = w - \eta \nabla_w \mathcal{L}(w, b)$$
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- The gradient: / Le gradient:

$$\nabla_w \mathcal{L}(w) = -\sum_{\forall c_n \neq \text{sign}(w^\top x_n + b)} c_n x_n$$
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- The updates / Les updates: (Same as the perceptron algo!/ Les memes updates qu'algo perceptron!)

$$w = w + \eta \sum_{\forall c_n \neq \text{sign}(w^\top x_n + b)} c_n x_n$$
$$b = b + \eta \sum_{\forall c_n \neq \text{sign}(w^\top x_n + b)} c_n$$

But can we do something better?

- Perceptron is basically / Perceptron est simplement:

$$f(x) = u(w^\top x + b)$$

where $u(.)$ is a step function (or sign(.) depending on how we define c_n) . / où $u(.)$ est un 'step function'.

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  - À cause de la step function on ne peut pas utiliser la modèle directement dans le fonctionne de cout.
- What if we want to have smooth, differentiable transitions? / Et si on veut avoir des transitions plus smooth?

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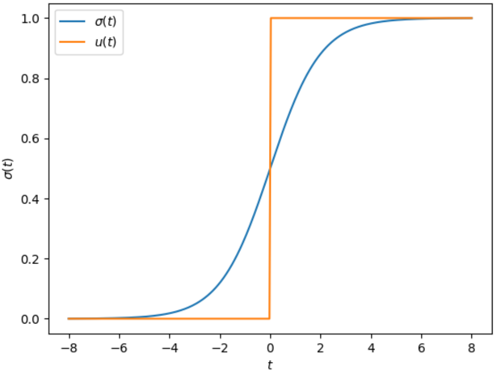
The logistic function

- We can use the logistic function $\sigma(t) = \frac{1}{1+e^{-t}}$. / On peut utiliser la fonction logistique.



Logistic regression

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- What will be the loss function? / Quelle sera la fonctionne de coût?
- How about the negative Bernoulli log-likelihood?

Bernoulli Distribution

- $\mathcal{B}E(y;\pi) = \pi^y(1-\pi)^{1-y}$, $y \in \{0,1\}$.
- Let's take the log / Prenons le logarithme

$$\log \mathcal{B}E(y;\pi) = y \log \pi + (1-y)\log(1-\pi)$$

.

- We will parametrize the Bernoulli distribution with $\pi = w^\top x + b$/ On est en train de parametriser la distribution Bernoulli.

Logistic regression loss function

- Training loss / La fonction de cout pour l'entrainement:

$$\mathcal{L}(w, b) = \sum_n \left(-y_n \log \pi_w(x_n) + (1 - y_n) \log(1 - \pi_w(x_n))\right)$$
$$= \sum_n \left(-y_n \log \left(\sigma(w^\top x_n + b)\right) - (1 - y_n) \log \left(1 - \sigma(w^\top x_n + b)\right)\right)$$

- Let's calculate the gradient with respect to w. / Calculons le gradient par rapport à w.

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- Few more things: $\frac{\sigma(x)}{dx} = \sigma(x)(1-\sigma(x))$, $\frac{\log x}{dx} = 1/x$.

The gradient wrt w

- Just do it! / Faisons-le!

$$\frac{d\mathcal{L}(w,b)}{dw} = -\sum_n y_n \frac{\sigma(w^\top x_n + b)}{\sigma(w^\top x_n + b)}(1 - \sigma(w^\top x_n + b))x_n$$
$$+ \sum_n (1 - y_n) \frac{1 - \sigma(w^\top x_n + b)}{1 - \sigma(w^\top x_n + b)} \sigma(w^\top x_n + b)x_n$$

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&+ \sum_n (1-y_n)\frac{1-\sigma(w^\top x_n + b)}{1-\sigma(w^\top x_n + b)}\sigma(w^\top x_n + b)x_n \\
&= -\sum_n y_n \frac{\cancel{\sigma(w^\top x_n + b)}}{\cancel{\sigma(w^\top x_n + b)}}(1-\sigma(w^\top x_n + b))x_n \\
&+ \sum_n (1-y_n)\frac{\cancel{1-\sigma(w^\top x_n + b)}}{\cancel{1-\sigma(w^\top x_n + b)}}\sigma(w^\top x_n + b)x_n
\end{aligned}
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&+ \sum_n (1-y_n)\cancel{\frac{1-\sigma(w^\top x_n + b)}{1-\sigma(w^\top x_n + b)}}\sigma(w^\top x_n + b)x_n \\
&= \sum_n \left(\sigma(w^\top x + b) - y_n\right) x_n
\end{aligned}
$$

- Makes a lot of sense! / Ça fait du sense!

The gradient wrt w

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$$\begin{aligned}
\frac{d\mathcal{L}(w,b)}{dw} &= -\sum_n y_n \frac{\sigma(w^\top x_n + b)}{\sigma(w^\top x_n + b)}(1-\sigma(w^\top x_n + b))x_n \\
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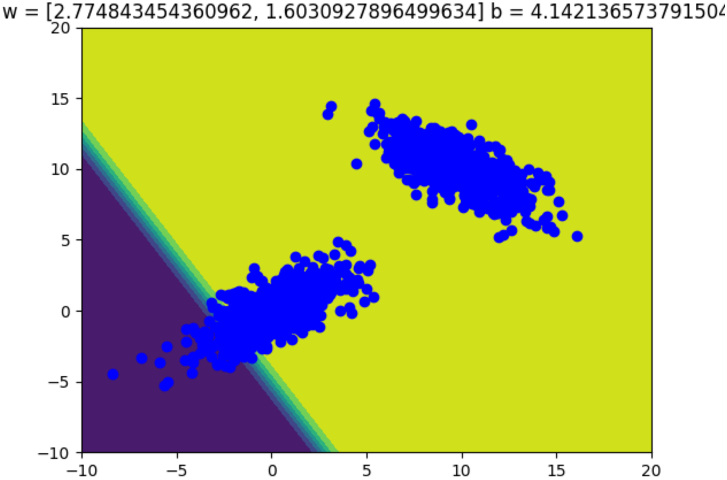
\end{aligned}$$

- Makes a lot of sense! / Ça fait du sense!
- torch will automatically do this for us. However, one needs to do this at least once in their life!
  - torch fait ça automatiquement mais on doit faire cette exercise au moins une fois dans notre vie!

Logistic Regression Epochs



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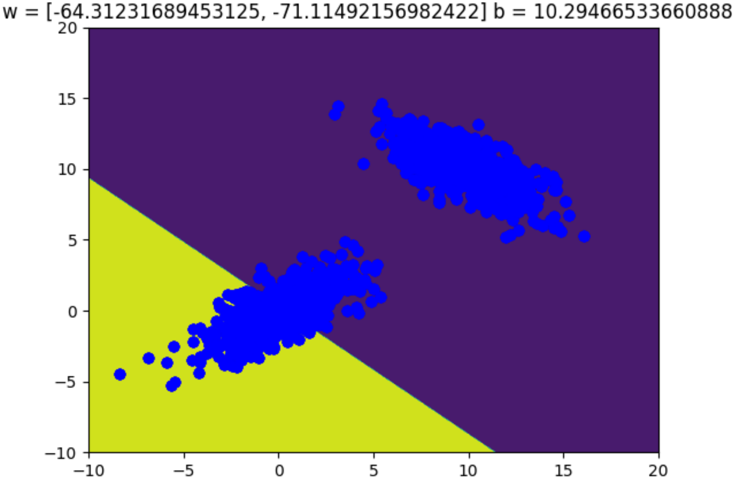

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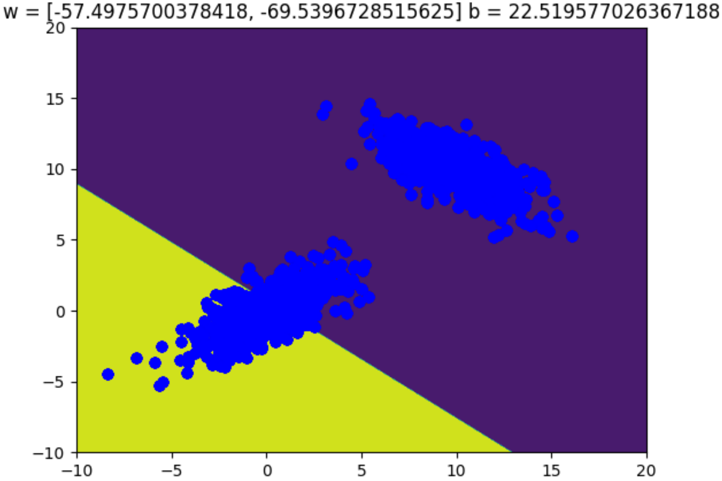

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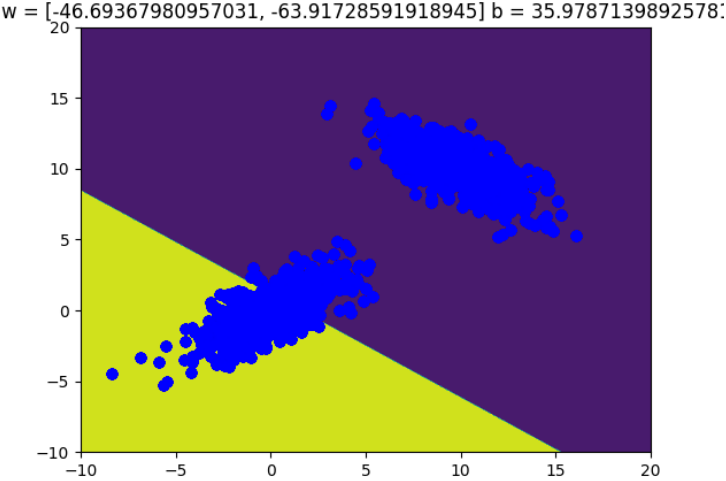

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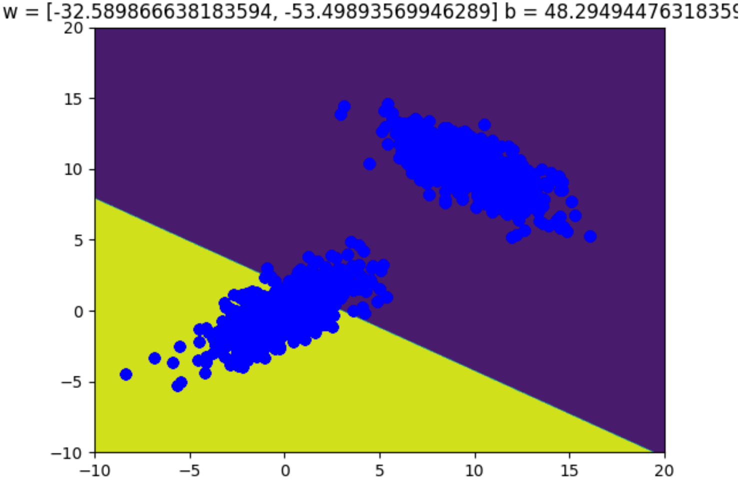

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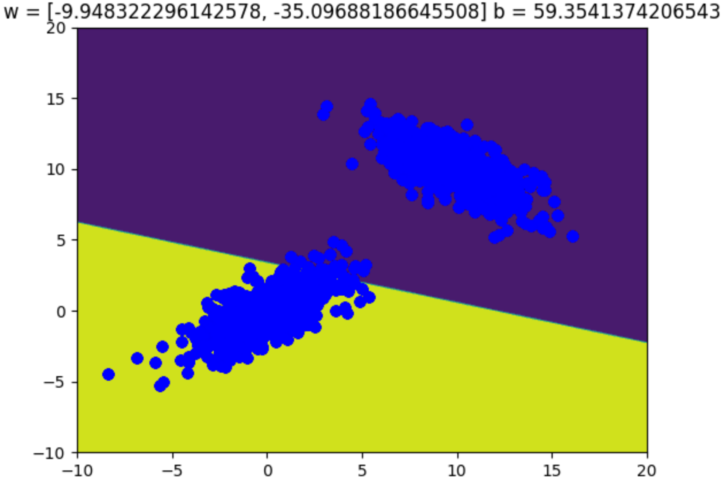

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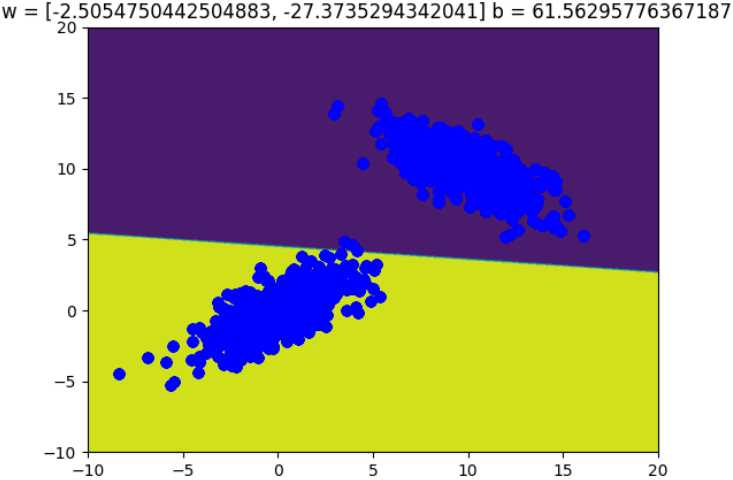

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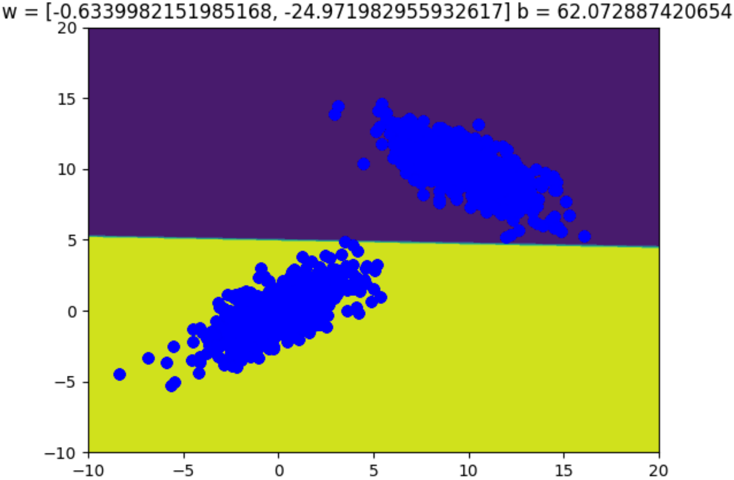

Generalizing to multiway classification

- Define (a new network!) / On défine une nouvelle réseau

$$\pi_k := \exp(w_k^\top x + b_k) / \sum_l \exp(w_l^\top x + b_l)$$

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- The loss function is now going to be the log-likelihood of discrete distribution / La fonctionne de cout sera maintenant le log-likelihood de le distribution discrète.

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&= \sum_n \sum_k [y_n = k]\left(w_k^\top x + b_k - \log \sum_l \exp(w_l^\top x + b_l)\right)
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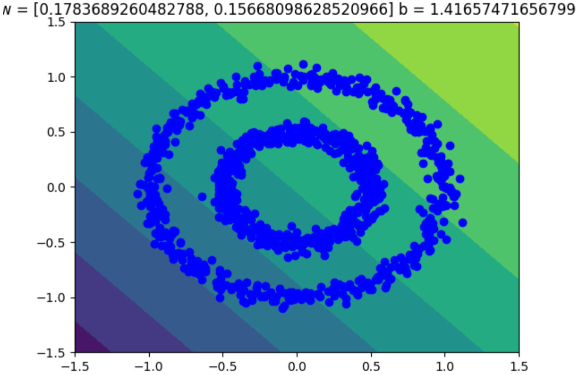
Non-Linear Classification

- What if we have something like this? / Qu'est-ce qu'on fait si on a ça?



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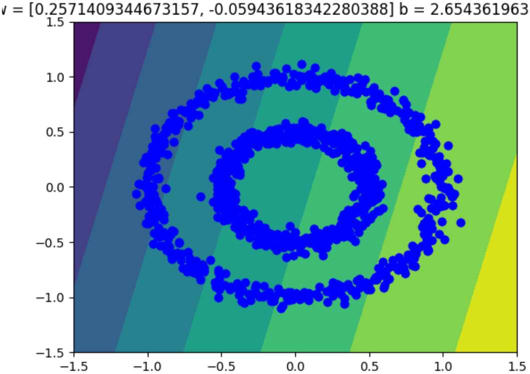

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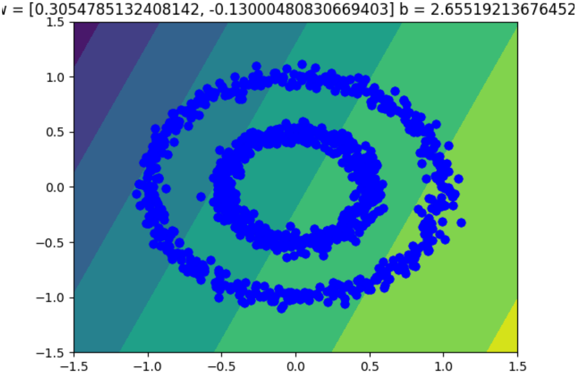

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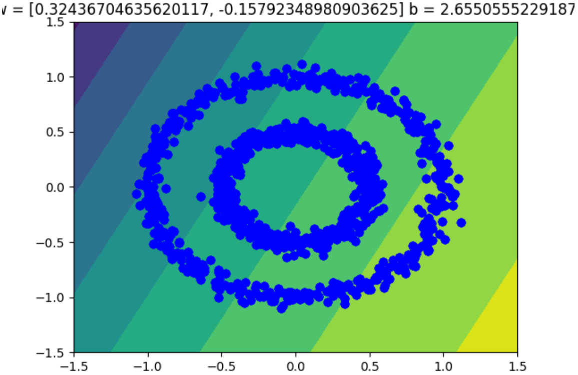

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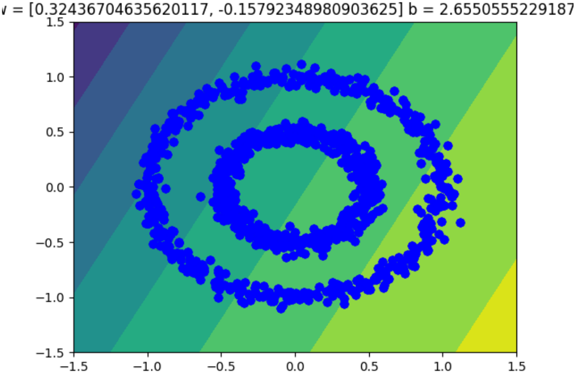

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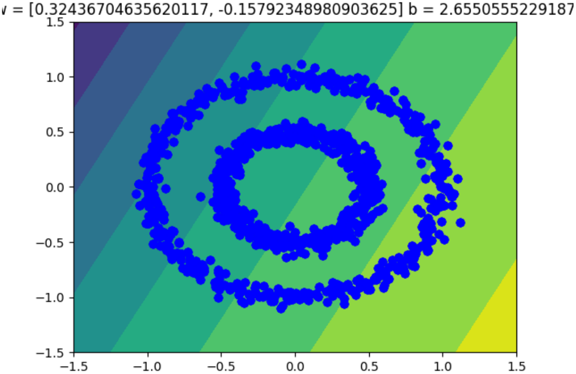

- Linear classifier is not enough! / Classificateur linéaire n'est pas suffisante!
- What can we do? / Qu'est-ce qu'on peut faire?

The Kernel Trick

- We can extend our linear classifier $y = w^\top x \rightarrow y = w^\top \phi(x)$. We introduce a feature transformation $\phi(.)$.
  - On améloire notre classificateur linéaire en introduisant une transformation de feature $\phi(.)$.

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\begin{aligned}
f(x) =& \sigma \left(\sum_n a_n \phi(x)^\top \phi(x_n) \right) \\
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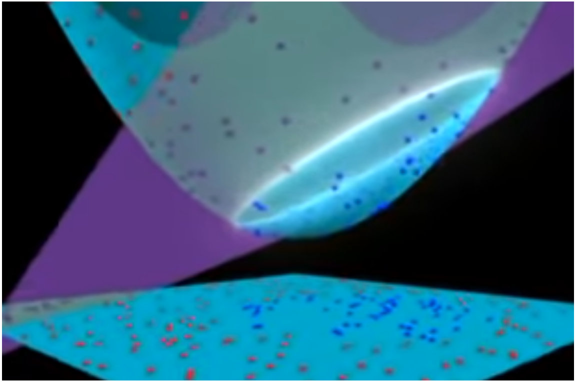
$$

- We can incorporate this in many places. But let's look at injecting this idea in logistic regression.
  - On peut incorporer cette idée dans différents place. Mais d'abord essayons d'injecter cette idée dans la regression logistique.

Motivating the Kernel Trick



Let's watch!

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Kernel Logistic Regression

- The coin bias is now estimated with the kernel-trick / Le biais est maintenant estimé avec le kernel trick.

$$\pi = \sigma\left(\sum_n a_n k(x, x_n)\right)$$

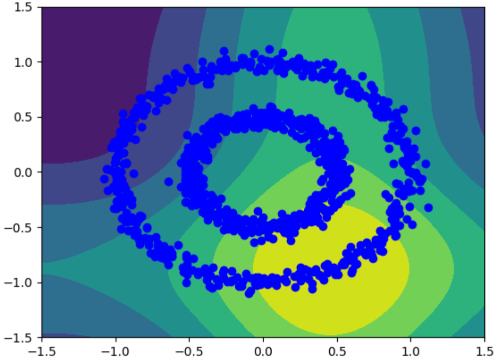
- Then the loss becomes, / le fonctionne de cout devient,

$$\begin{aligned}\mathcal{L}(a) = \sum_n \Big(& - y_n \log\left(\sigma\Big(\sum_j a_j k(x_n, x_j)\Big)\right) \\ & - (1 - y_n) \log\left(1 - \sigma\Big(\sum_j a_j k(x_n, x_j)\Big)\right)\Big)\end{aligned}$$

- Kernel functions are chosen from options such as RBF Kernel, Polynomial Kernel,.. / On choisit le kernel entre les options qu'on est habitué.

$$k_{\text{rbf}}(x, x_n) = \exp(-\gamma \|x - x_n\|_2)$$

Kernel Logistic Regression Learning Steps



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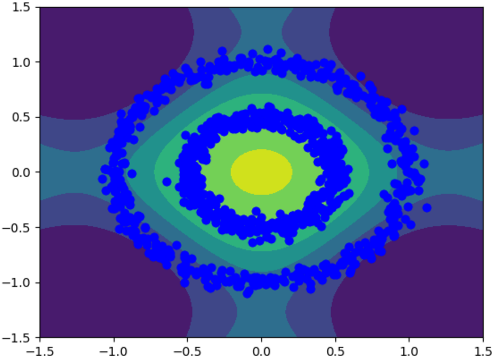

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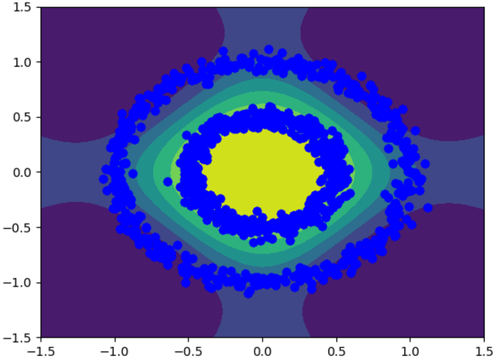

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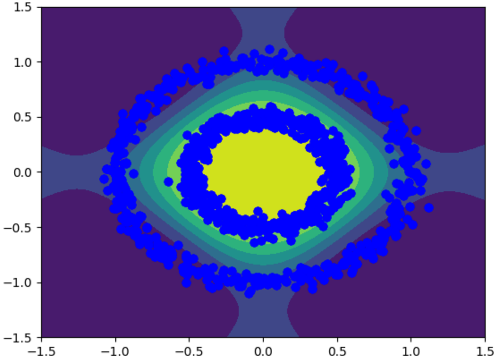
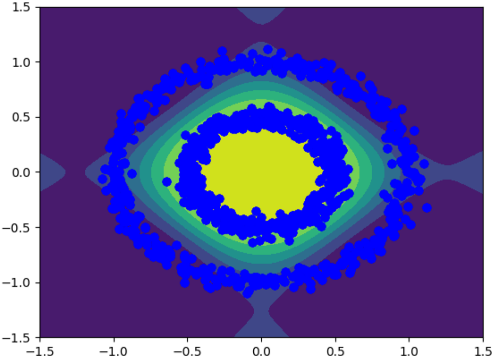


Kernel Logistic Regression Learning Steps



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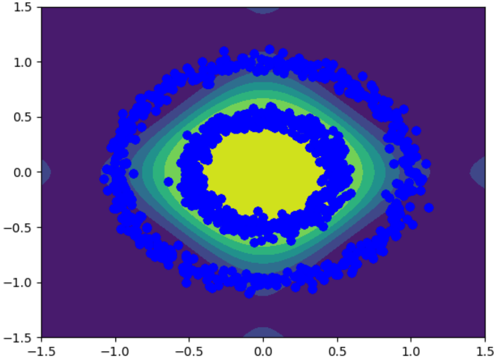

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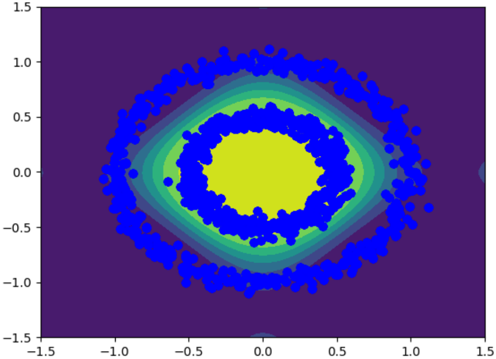

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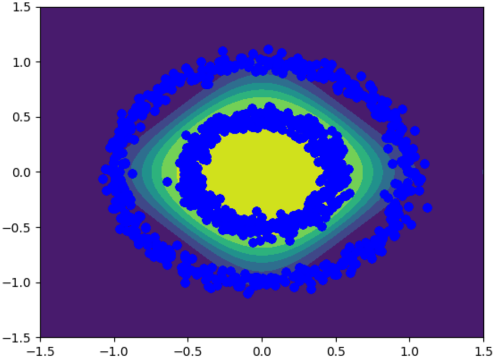
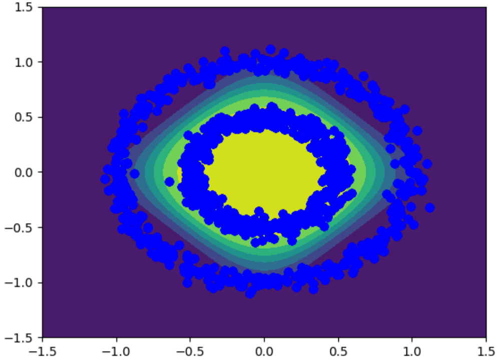


Kernel Logistic Regression Learning Steps



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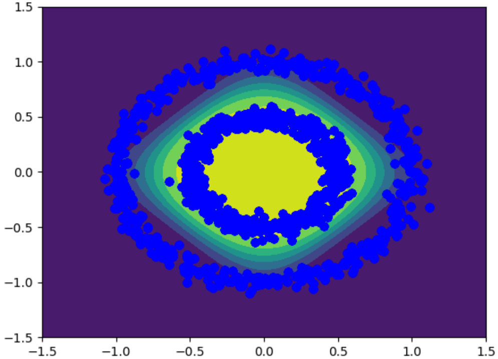

Table of Contents

Neural Network Classification

- The bias parameter can be estimated with a general function $f(x)$.

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\begin{aligned}
\pi =& f(x) \\
=& \sigma(w_M^\top h(W_{M-1}^\top h(W_{M-2}^\top(\dots h(W_1^\top x)))))
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- The general function maps x from L dimensions to K_1, K_2, $\dots$, K_{M-1} dimensions. The last layer w_M is a vector and maps to 1 output dimension. / Chaque couche map à une dimensionnalité différent K_l.

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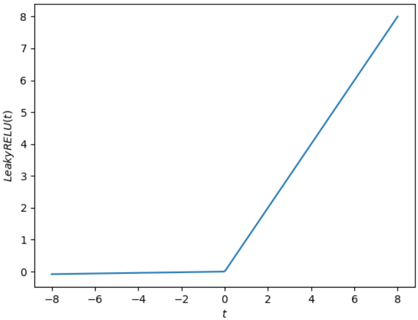
- The loss function becomes / La fonction de cout devient:

$$\mathcal{L}(\theta) = \sum_n \Big(-y_n \log\Big(\sigma\Big(f_\theta(x_n)\Big)\Big) - (1-y_n)\log\Big(1-\sigma\Big(f_\theta(x_n)\Big)\Big)\Big)$$

An Example Neural Network

- $f_\theta(x) = w_2 h(W_1^\top x)$, $h(x) = \text{LeakyRELU}(t)$, $W_1 \in \mathbb{R}^{2\times 100}$, $w_1 \in \mathbb{R}^{100}$.



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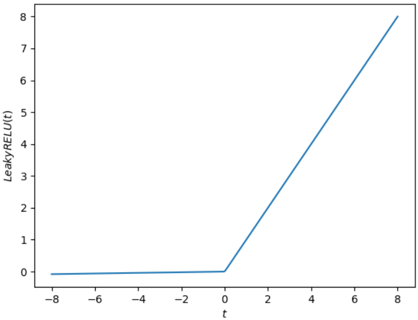

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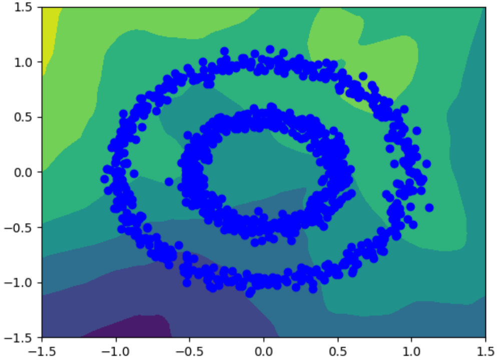
- Let's say we define $\phi_{W_1}(x) := h(W_1^\top x)$.

Neural Network Classifier Learning Steps



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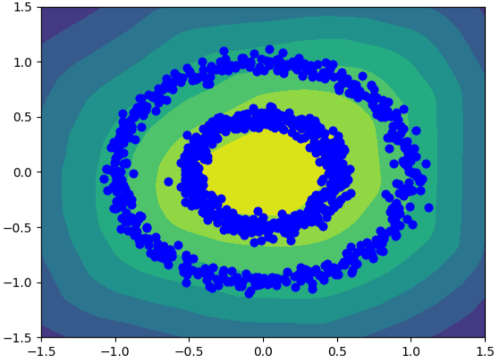

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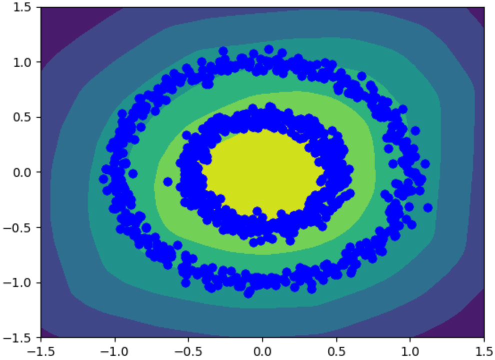

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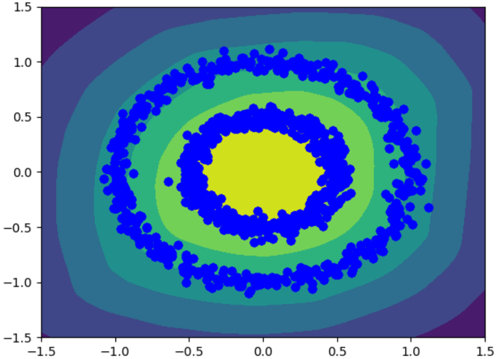

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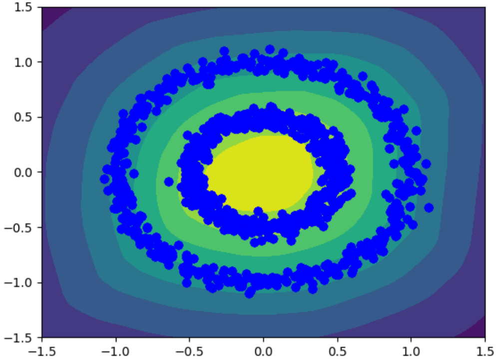

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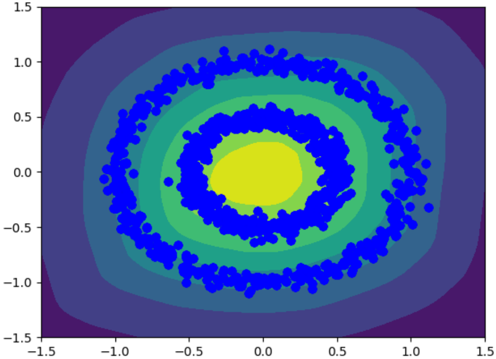

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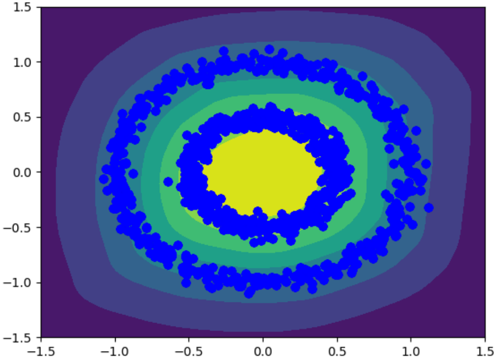

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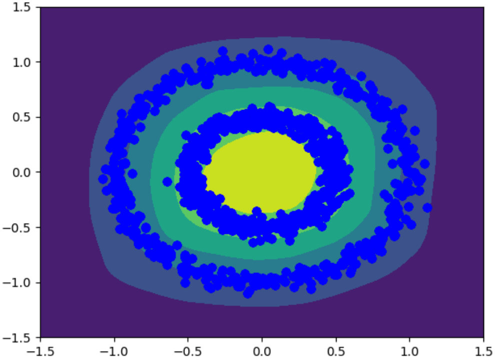

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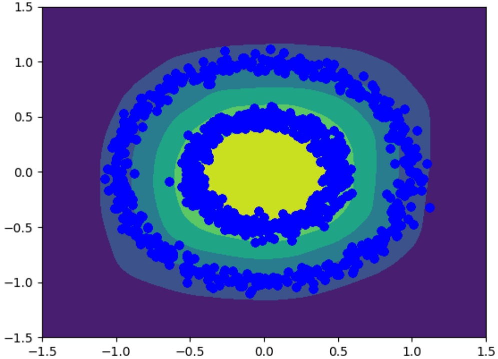

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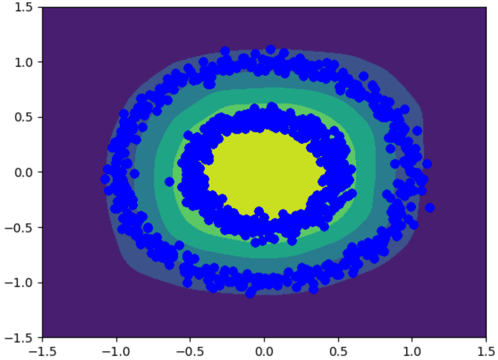

Kernel Methods vs Neural Networks

- Kernel Methods are not very suitable for large datasets because of the hard dependency on the training set $x_1, \ldots, x_N$. / Les méthodes de noyau ne sont pas très adaptes pour des datasets larges parce qu'ils dépendent sur le training set.
- Kernel Methods come with an interpretability advantage. At the end of the day it's a linear model, and we have the importance weights. / Les méthodes de noyaux sont plus interpretables car ils sont linéers.
- Neural Nets are more modern because of the way we train them. They more amenable to train with SGD. / Les résaux de nérones sont plus adaptes à entrainer avec SGD sur des grands jeux de données.

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- We have not talked about SVMs. It's an important idea to know, but you can get that from many other classes. On n'a pas parlé de SVMs.

Suggested reading

- Bishop, chapters 4, 5, 6

Next week

- More deep learning!