IFT 4030/7030, Machine Learning for Signal Processing

Week3: Signal Processing Primer

Cem Subakan

Admin

- How was the first lab?
  - Comment était le lab 1?
- The next lab is on friday!
  - On aura le deuxième lab en vendredi.
- How are the project proposals coming along? The proposals are due the second week of october.
  - Comment va la réflection sur les projets? Les propositions de projets sont dûs la deuxième semaine de l'octobre!

Today

- Signals/Time series, what are they?
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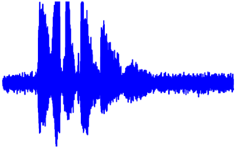
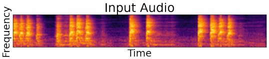
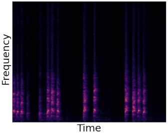
  - Comment est-ce qu'on répresente les signaux?
    - Time Domain / Frequency Domain / Time+Frequency Domain







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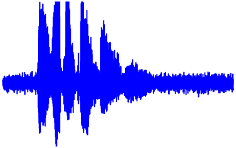

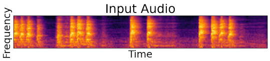

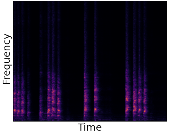

- The Fourier Transform
  - The Short-Time Fourier Transform

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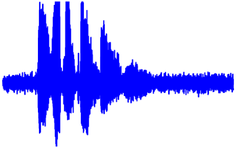

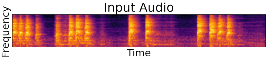

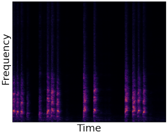

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  - Convolution

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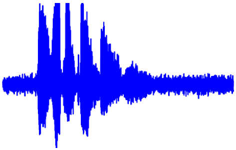
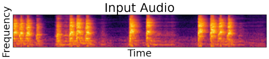
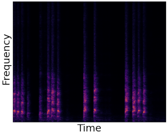


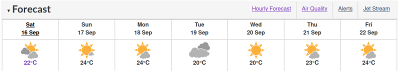


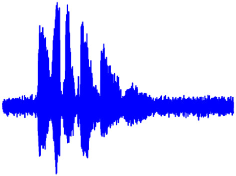


- The Fourier Transform
  - The Short-Time Fourier Transform
- Filtering / Convolution
  - Convolution
- Sampling
  - Échantillonnage

Signals

- A dry definition: A signal is an ordered collection of numbers
  - Une définition sec: Un signal est une collection des chiffres en ordre.





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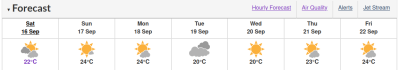

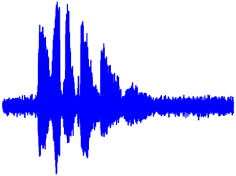

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Representing Signals/Time Series

- There are different ways of representing time series/signals.
  - Il existe plusieurs façons de répresenter les signaux.
- Each representation type has its pros / cons.
  - Chaque type de représentation a leurs avantages / desavantages.

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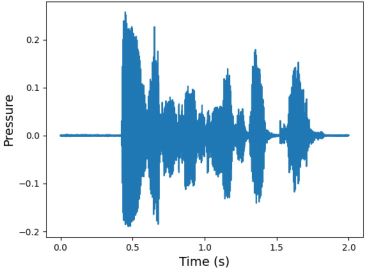
- Some typical options: Time Domain, Frequency Domain, Time+Frequency Domain

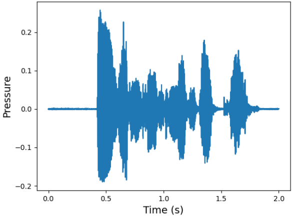
Sound as Signal

- We will start with sounds as an example, but any time series would do.
  - On commencera avec les sons comme un exemple, mais n'importe quel time series serait ok.
- Let's `listen`.



The time domain

- It's kinda difficult what to take out of this by looking at it.
  - Je sais pas quoi faire en regardant ça.



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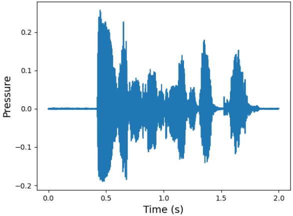

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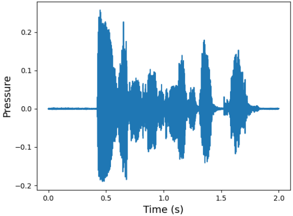

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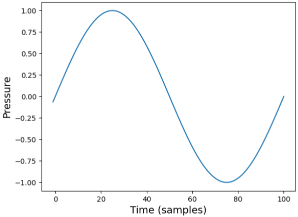
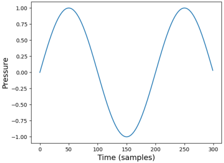
- Sinusoids!! (They are special)

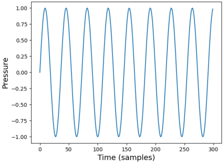


Sinusoids

- 80 Hz, `listen`.



- 440 Hz, `listen`. This is how we tune our instruments.
  - On tune nos instruments à La 440Hz.



- 880 Hz, `listen`
- 1760 Hz, `listen`
- 15000 Hz, `are you able to hear this at all?`
  - Pouvez-vous entendre ceci?

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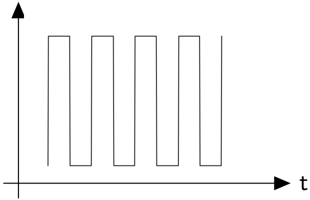
Decomposing a signal

- Let's decompose a square wave with sines.
  - Decomposons un square wave.



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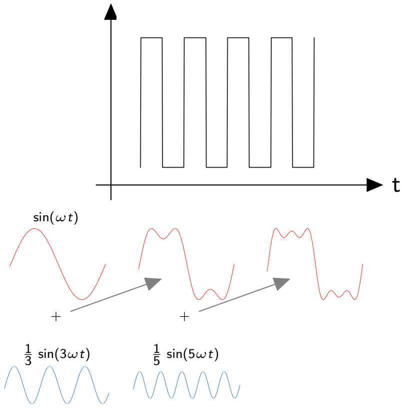

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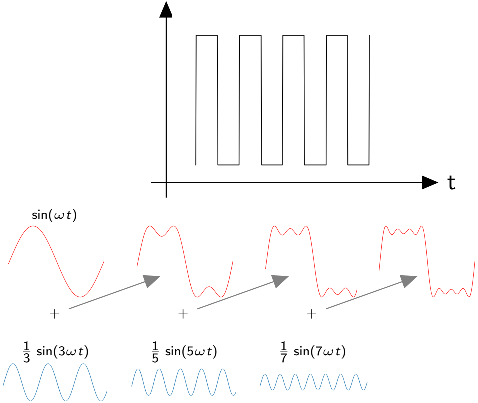

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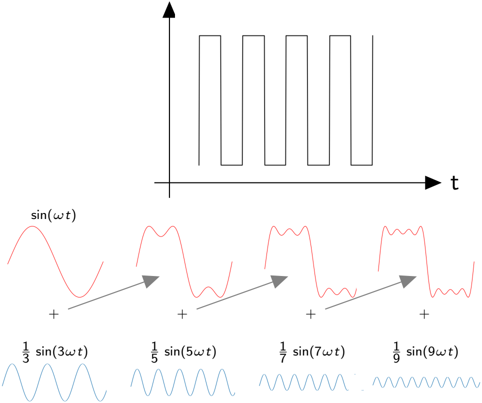

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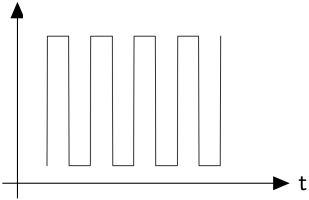

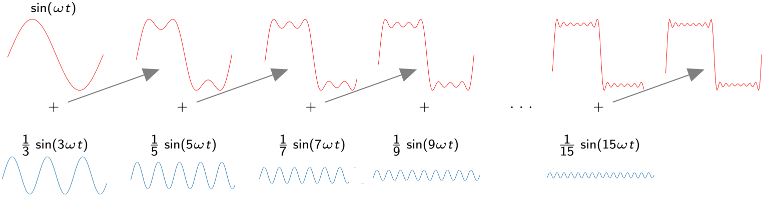

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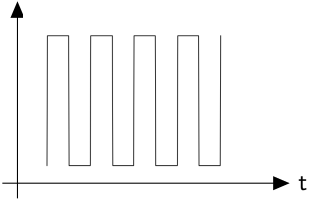

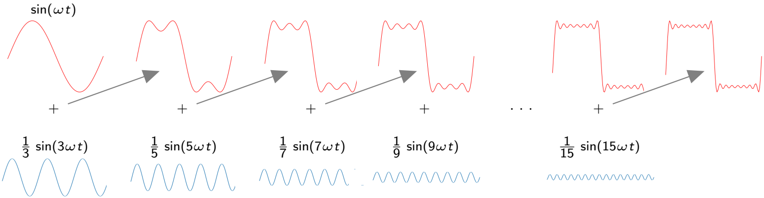

- So we can approximate a square wave with
  - Alors on peut approximer un onde carré avec

$$SW(\omega t) \approx \sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \frac{1}{7}\sin(7\omega t) + \dots$$

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- **Inner Product!**

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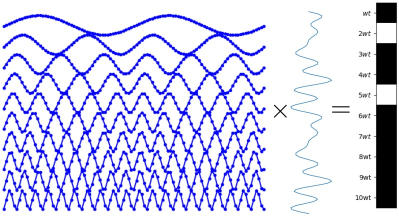

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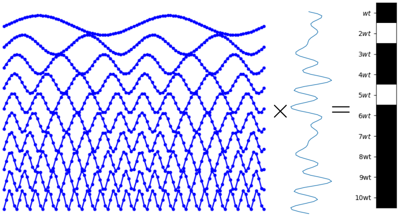

- This is nice, but is this general enough?
  - Bon, mais est-ce assez générale?

No! You need cosines to span the space

- The signal we saw before was,
  - Le signal qu'on avait vu était:

$$y(t) = \sin(2\omega t) + \sin(5\omega t) + \underbrace{\cos(6\omega t)}_{\text{We missed this guy!}}$$

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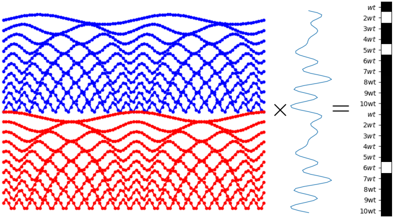

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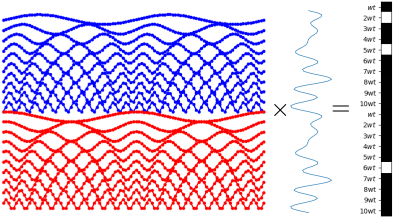

- But wait, I remember that Fourier Transform gave us complex numbers. What's that about?
  - Mais chuis confus-là, je me souviens que FT nous donnait des chiffres complexes?

Fourier Transform Formal Definition

- Remember Euler's formula:

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- So it seems like, we can use complex exponentials to project on to cosines and sines at the same time.
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- Discrete Fourier Transform (DFT):

$$X_k = \sum_{n=0}^{N-1} x_n \exp\left(-j2\pi \frac{k}{N} n\right), \text{ where}$$

$$\mathrm{x} = [x_0, x_1, \ldots, x_{N-1}],\ k \in \{0, \ldots, N-1\}.$$

- Note that k corresponds to frequencies, and we have the same number of frequencies as the signal length N.
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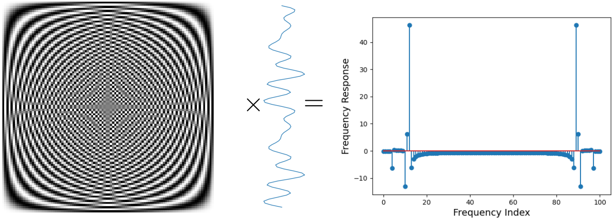
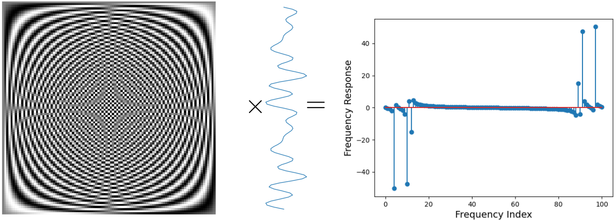
- By the way, do you see that this is a matrix? $\exp\left(-j2\pi \frac{k}{N} n\right)$
  - Vous voyez que c'est une matrice?

DFT in action

- Real Part / La partie réel



- Imaginary Part / La partie imaginaire



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  - Notez que les résultats sont symmetriques. C'est à cause de la construction de la matrice DFT.
- Note that the DFT matrix repeats after the half. We will get back to this later.
  - Notez que les matrices DFT répetent après la moitié. On va parler de ça après.

Another way of interpreting DFT

- Note that we get complex numbers. We can calculate magnitude and phase.
  - Notez qu'on obtiens des nombres complex. Donc on peut calculer la magnitude et la phase.

$$
\begin{aligned}
|X_k| &= \sqrt{\text{Re}(X_k)^2 + \text{Im}(X_k)^2} \\
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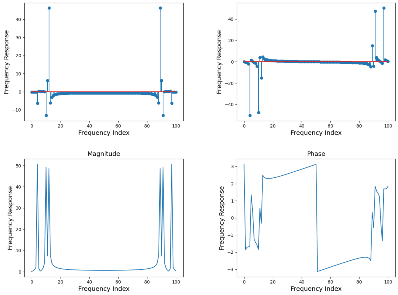

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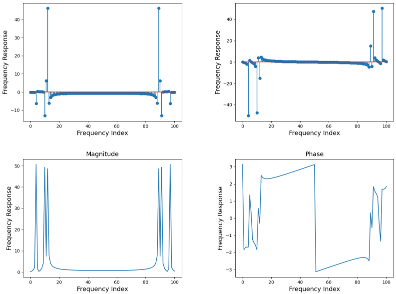
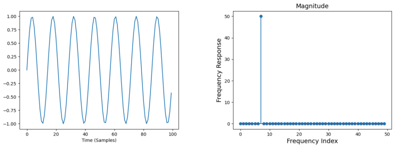


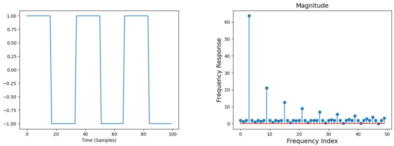
- Magnitude is easy to interpret. Phase is not as easy always.
  - Magnitude est facile est interpreter. La phase n'est pas toujours si facile à interpreter.

DFTs of different signals

- A single sinusoid



- Square wave



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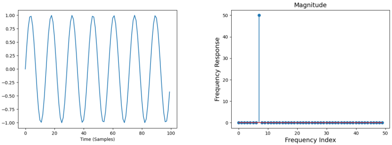

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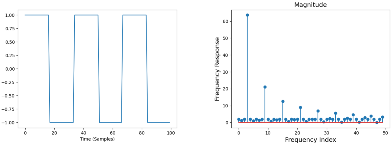
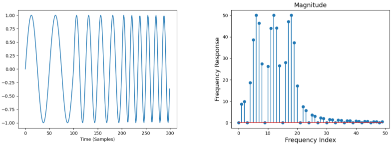


- Sinusoids with increasing frequencies



- So, for time varying signals, DFT is not that great.
  - Pour les signaux qui varie avec le temps DFT n'est pas idéal.

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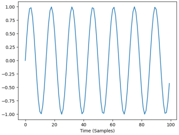
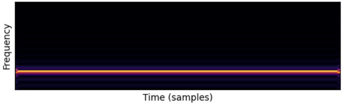
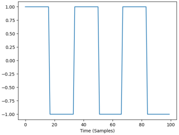
Time-frequency representation

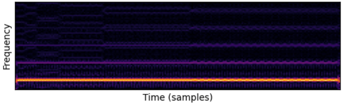
- A single sinusoid





- Square wave





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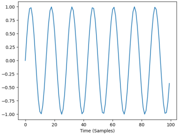

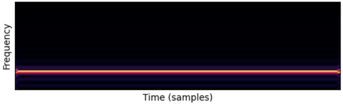

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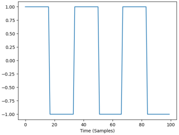

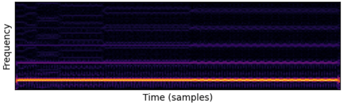

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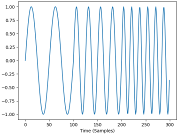
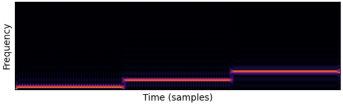
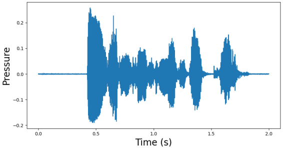




A real example for speech

Time Domain



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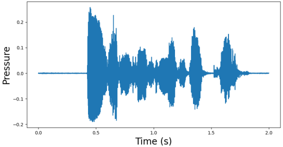

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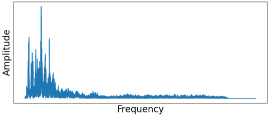

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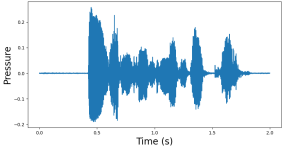

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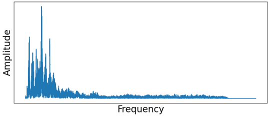
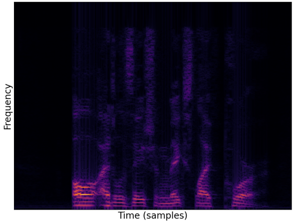
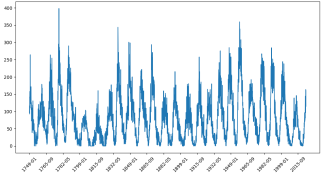


Time-Frequency Domain – We 'see' the signal. – on voit le signal.



An example from another domain

Time Domain
Counts of sunspots wrt. time



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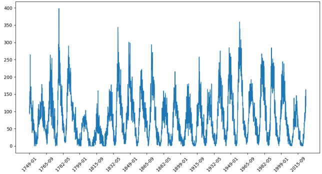

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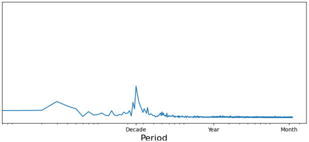

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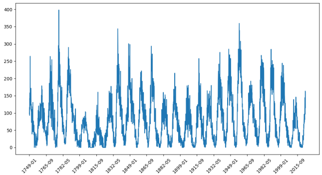

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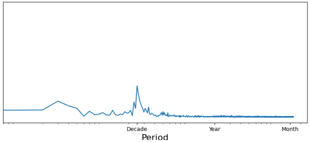

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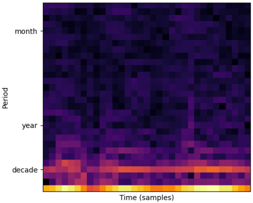
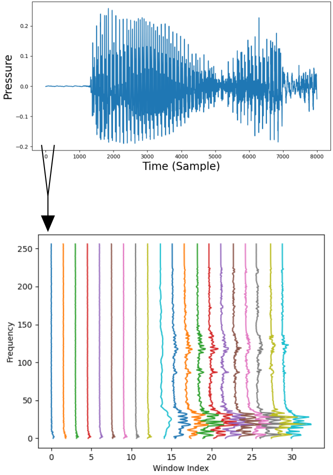


How to Calculate the Spectrogram (STFT)



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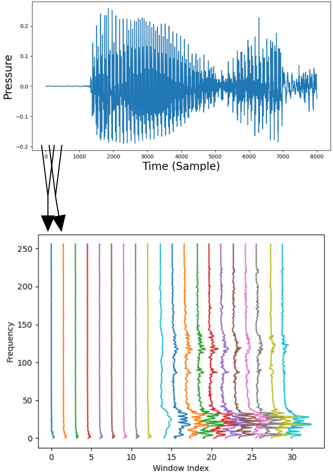

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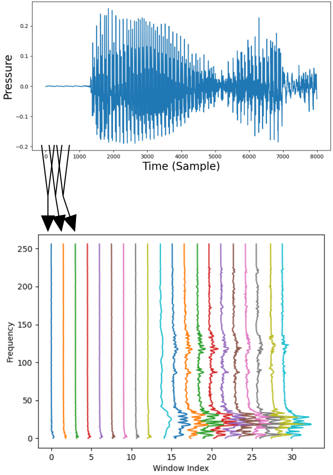

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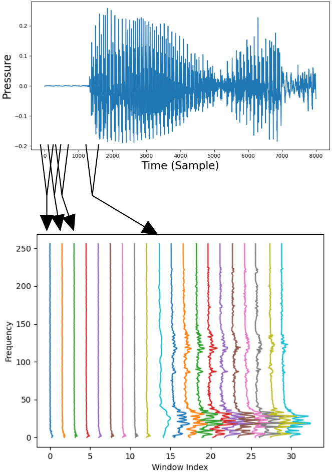
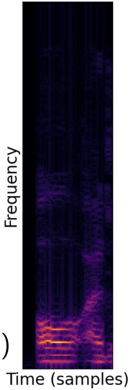
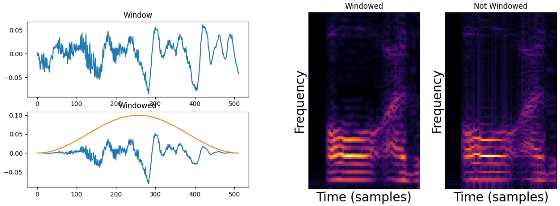


It’s nice to show it as an image! (imshow())



STFT considerations

- Windowing (Utilisation de fenetres)
  - We apply a window, before calculating the DFT.
    - On applique une fenetre avant de calculer le DFT.



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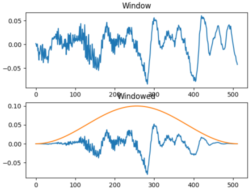

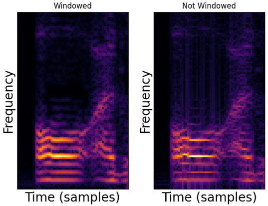

- Notice that not windowed STFT contains more high frequency artifacts
  - Notez que STFT qui n'est pas windowé contiens plus des artifacts de haute fréquence.

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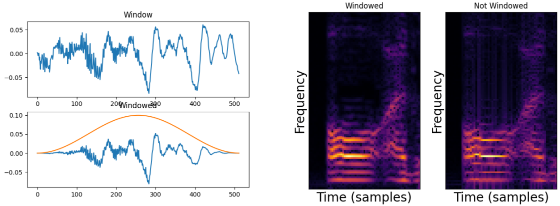
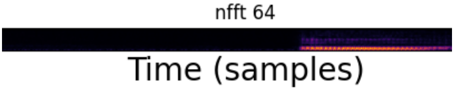
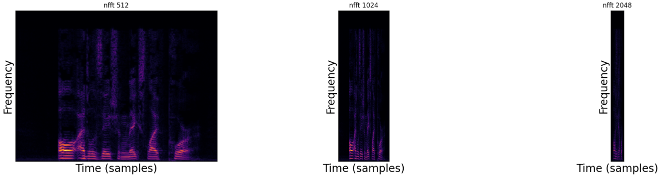


- Notice that not windowed STFT contains more high frequency artifacts
  - Notez que STFT qui n'est pas windowé contiens plus des artifacts de haute fréquence.
- We add an overlap to make up for using tapering windows. / On ajoute un overlap aussi pour compenser pour les fenetres qui réduit en amplitude.

Time/Frequency Tradeoff

- Heisenberg's uncertainty principle - We can not know the frequency and time location of a wave.
  - On ne peut pas déterminer la fréquence et et la localisation temporelle d'une onde.
- In the context of spectrograms: Big DFT sacrifice temporal resolution, Small DFTs have bad frequency resolution
  - Grand DFT a une mauvaise resolution temporelle, Petit DFTs ont une mauvaise resolution fréquentielle





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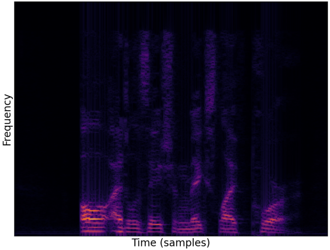
Mel-Frequency Spectrograms

- You notice that most of the energy is concentrated on the lower frequencies.
  - Vous vous rendez compte que l'energie est plutot concentrée dans les frequences bas.



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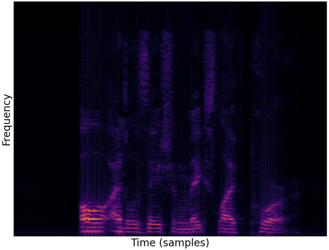
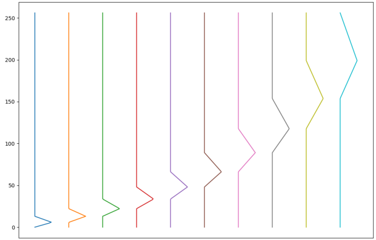


- This is a bit wasteful.

Mel-Frequency Spectrograms

- We can 'warp' the frequency axis.



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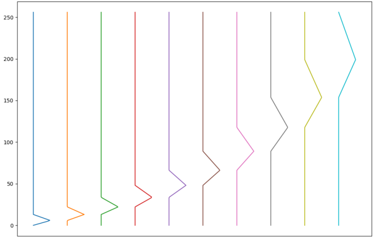
$\sum_f$



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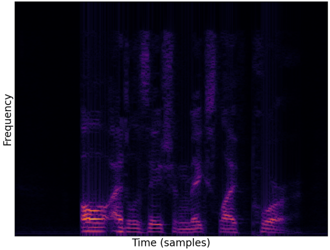

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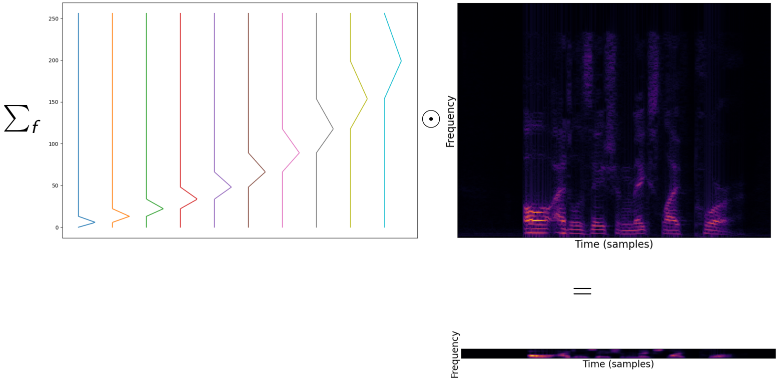

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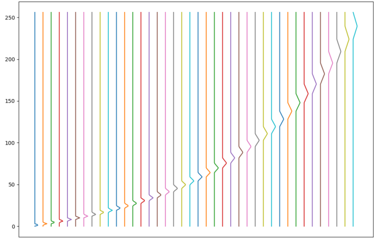
- We can ‘warp’ the frequency axis with more filters also (e.g. 40).



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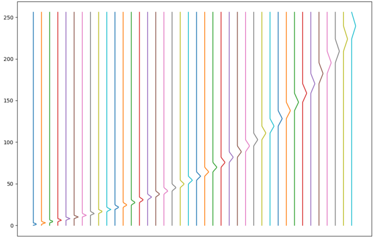

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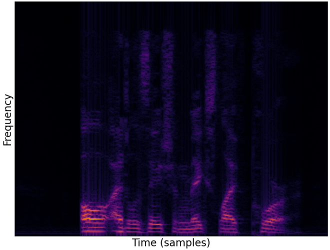

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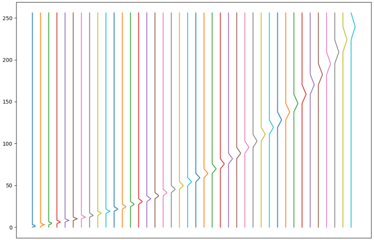

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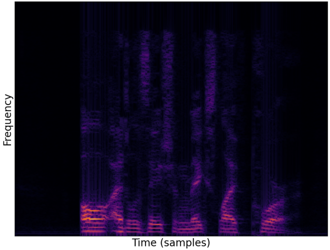

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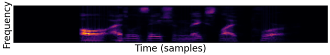
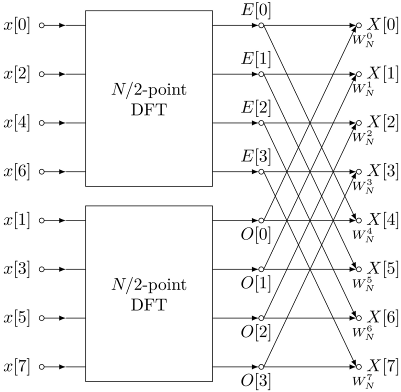


Fast Fourier Transform (FFT)

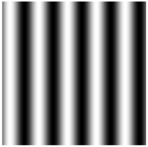
- DFT matrix has symmetries.
- DFT can be decomposed into DFTs of half the size.
  - La DFT peut etre decomposé aux 2 DFTs avec la moitié de la taille originale.
- We reduce the complexity from $\mathcal{O}(N^2)$(*why*?) to $\mathcal{O}(N \log N)$.

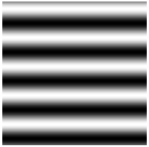


- Whenever you can use FFTs, use em!

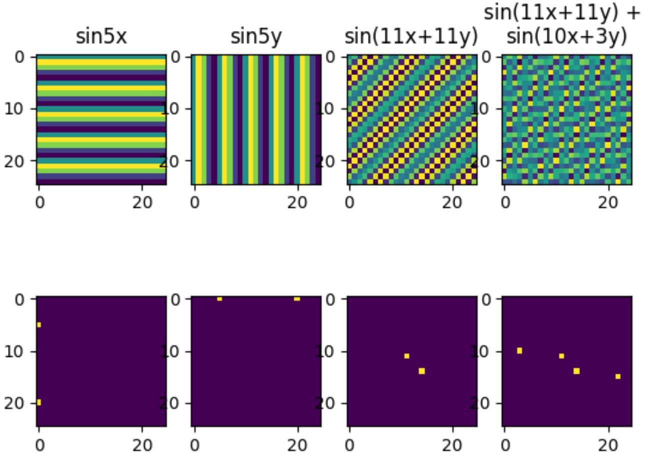
Image DFTs

- We can généralize to images as well
  - On peut généraliser aux images aussi.
- The DFT bases in 2d
  - Les bases DFT en 2d





Some example DFTs



How to calculate 2d DFTs?

- For images $Y = FXF$
- For Tensors $F_{il_1} X_{ijk} F_{jl_2} F_{kl_3} \rightarrow Y_{l_1, l_2, l_3}$

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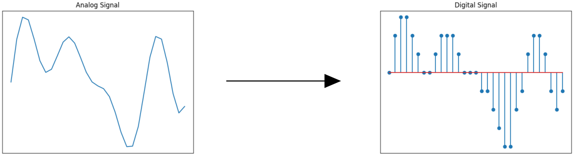
Analog to Digital

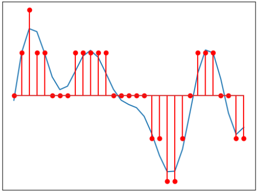
- How do we store / acquire signals?
  - Comment est-ce qu'on mets les signaux dans les ordinateurs?
- We convert the analog signals to digital.
  - On convertis signaux analogues au digital.



- It's not straightforward how to do this conversion.
  - C'est pas trivial comment faire cette conversion.

Signal Representation – Quantization

- We have to quantize a signal in order to digitize. (Quantize the y-axis)
  - On est obligé de quantiser pour digitiser. (l'axe y)



- We measure the precision in terms of bits. More bits we use, more signal-to-noise ratio we have.
  - On mésure la precision de la quantization en terme de bits. Plus de bits qu'on utilise, ça donne plus de SNR.

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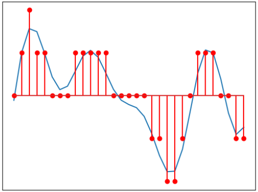

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  - On mésure la precision de la quantization en terme de bits. Plus de bits qu'on utilise, ça donne plus de SNR.
- Example average numbers:
  - 8bit - 48dB poor
  - 12 bits - 72dB okayish
  - 16 bits - 96 dB good
  - 24 bits - 144 dB overkill

Quantization in practice

8, 7, 5 bits

4, 3, 2 bits, 1 bit

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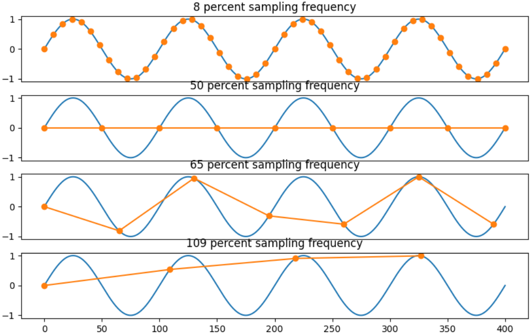
Sampling / Échantillonnage

- How often we sample is also important!
  - C'est aussi important la fréquence avec la quelle on échantillon. (l'axe de temps)
    - We use Hz to measure sampling freq. / On utilize Hz pour mésurer la fréquence d'echantillonnage.
- Important: Nyquist Rate: We must sample x2 above the highest frequency we want to represent.
  - Important: Le rate de Nyquist: On doit échantillonner 2 fois au délà de la fréquence qu'on veut répresenter.
- Perceptual Limits:
  - Our ears: We hear up to 20kHz (declines with age) So above 40kHz sampling is required.
  - Seeing: We perceive only up to 60Hz, so 120 Hz or up is required.
- Limites perceptuels:
  - Nos oreilles: La limite est 20kHz. Donc on a besoin d'une fréquence d'échantillonnage de 40Hz.
  - Nos yeux: La limite est 60Hz. On a besoin donc 120Hz.
- Common sampling rates:
  - Speech: 8kHz, 16kHz, Music: 32kHz, 44.1kHz, Pro-Audio 96kHz
  - Movies: 24fps (recently 48fps) HDTV: 60fps, ...

Aliasing

- Frequencies above 50% of the sampling rate are mis-represented.
  - Les fréquences sur 50% du rate d'échantillonnage sont mal représentés.



In real-life

- Electricity consumption sampled poorly.
  - Une mauvaise échantillonnage de la consomptions de l'éléctricité.

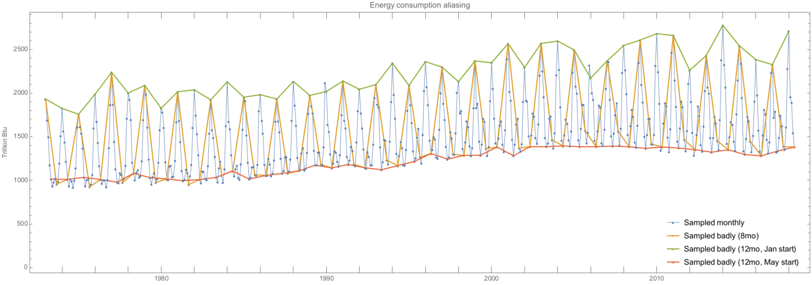


image taken from UIUC MLSP

In real-life

- Electricity consumption sampled poorly.
  - Une mauvaise échantillonnage de la consomptions de l'éléctricité.
- Different sampling leads to different conclusions.
  - Différent l'échantillonnage mène à des conclusions différentes.

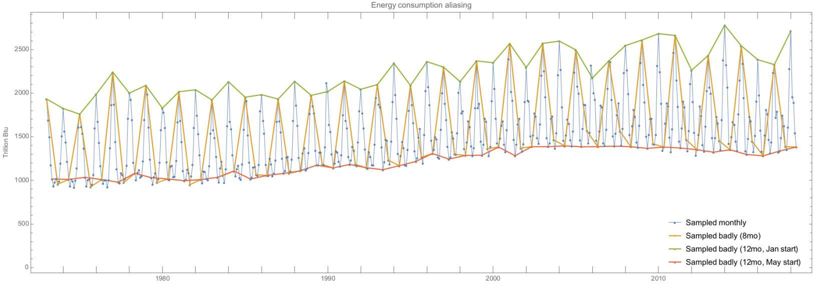
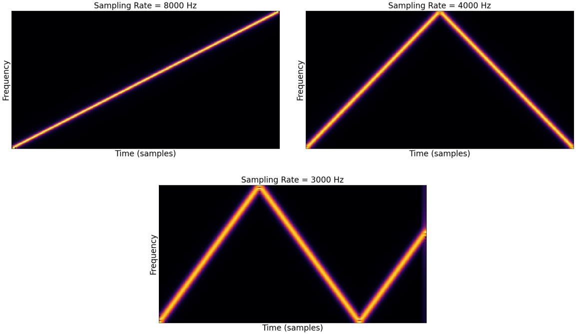


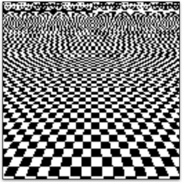
image taken from UIUC MLSP

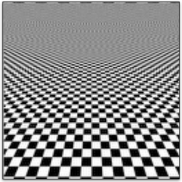
Aliasing in Sinusoid Sweeping



We have frequency sweeps from 0-4kHz, for different sampling rates.
Notice the aliasing when sampling rate is lower!
`Sweep 1` `Sweep 2` `Sweep 3`

Aliasing in Images/Video





https://www.youtube.com/watch?v=R-IVw8OKjvQ

Table of Contents

Convolution

- This is an extremely important operation that comes up all the time. (e.g. filtering)
  - C'est une opération extremement importante qu'on va voir souvent.

$$
\begin{aligned}
x(t) * w(t) := & \sum_{i=0}^{M-1} x(i) w(t-i) \\
= & x(0)w(t) + x(1)w(t-1) + x(1)w(t-2) + \dots \\
& + x(M)w(t-M)
\end{aligned}
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&+ x(M)w(t-M)
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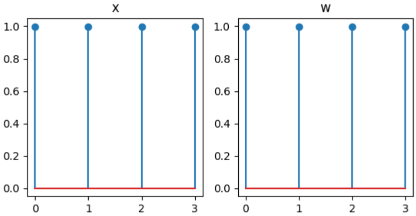
$$

- If x is of length M and w is of length N, the result is of length $M + N - 1$.
  - Si x est de longeur M et w est du longeur N, le résultat est de longeur $M + N - 1$.

Convolution

$$y = x * w$$



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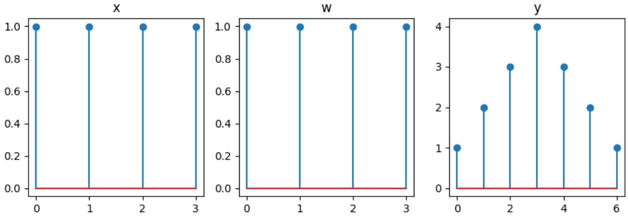

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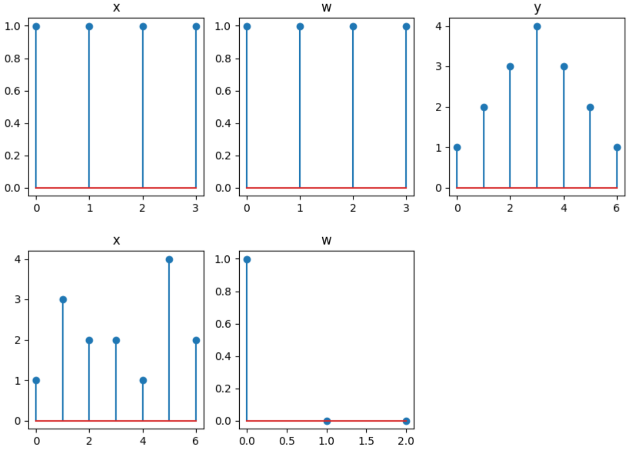

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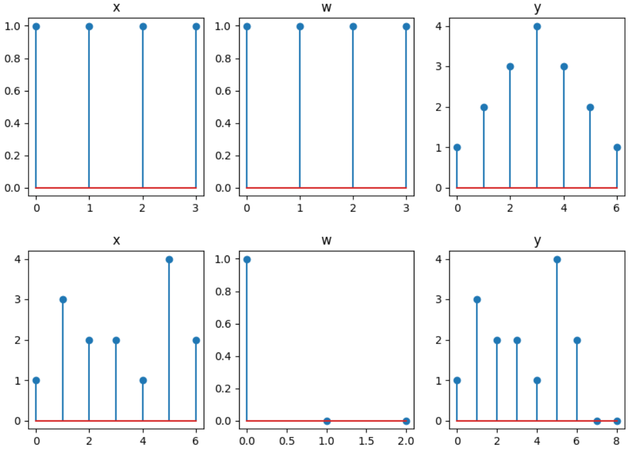

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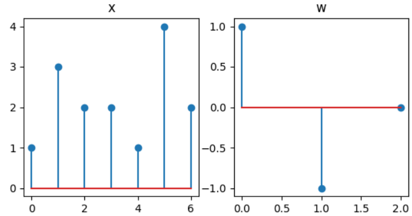

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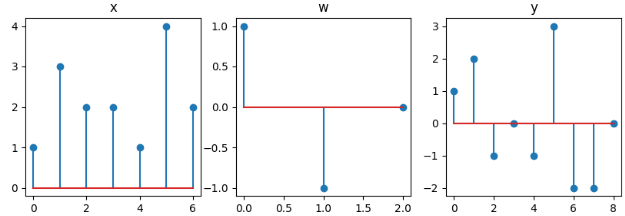

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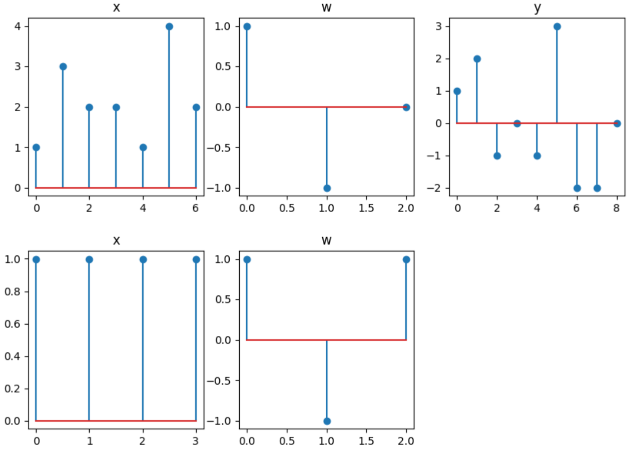

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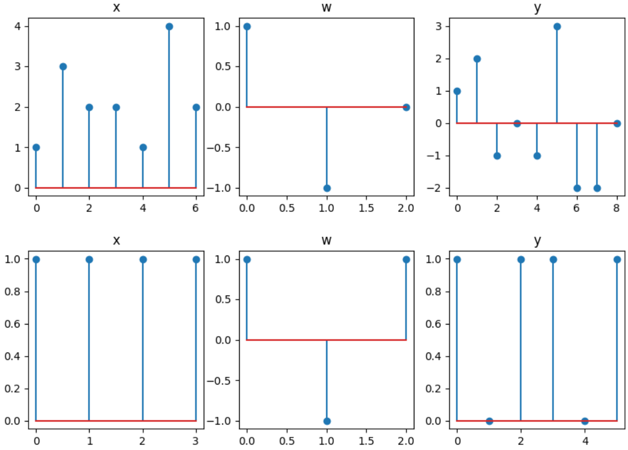
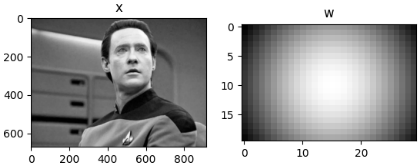


Image Convolution

$$y = x * w$$



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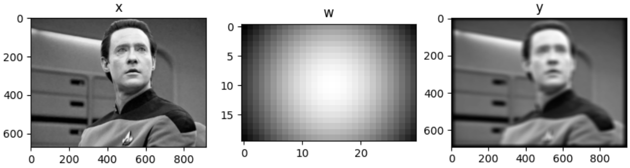

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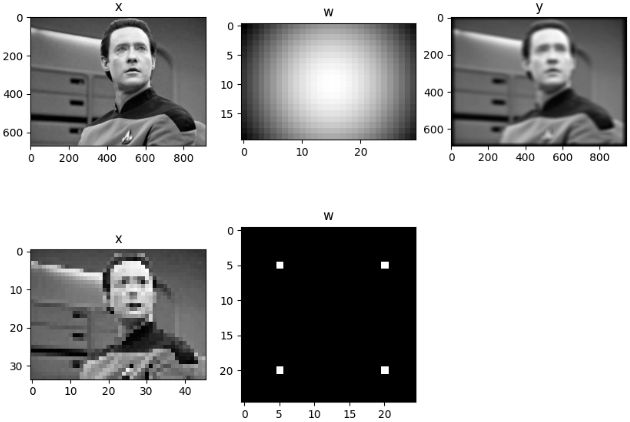

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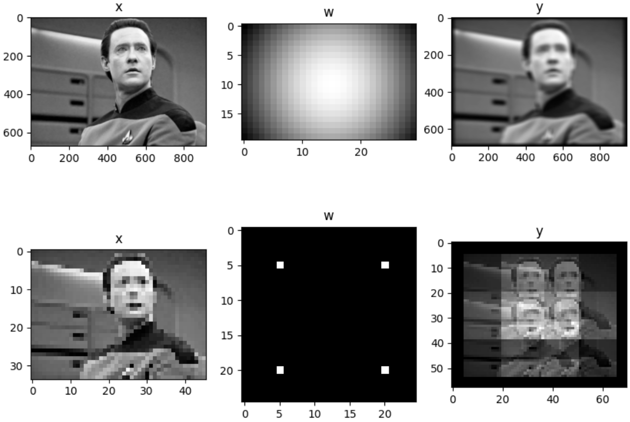
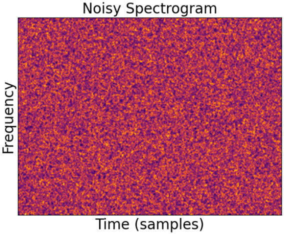


Filtering Audio with Convolution

- Consider this noisy audio. `Listen`.
- The filtered version with an averaging kernel. `Listen`.
  - La version filté avec un noyau de moyenne.



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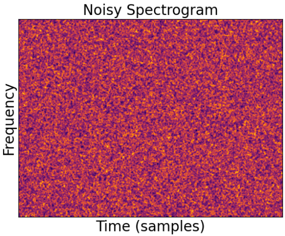

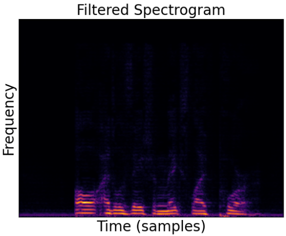

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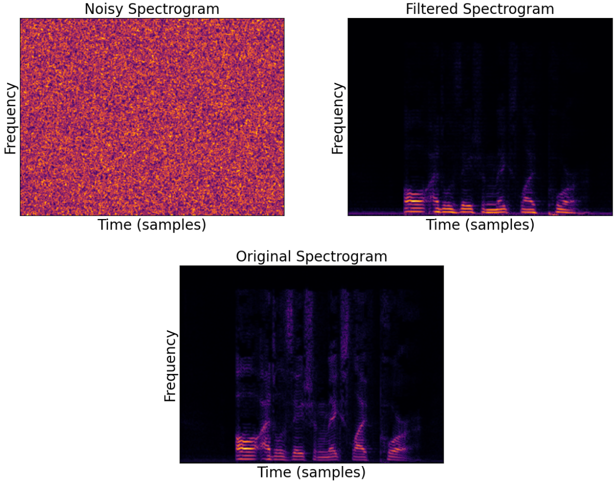
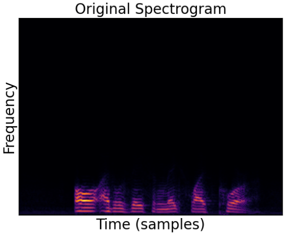
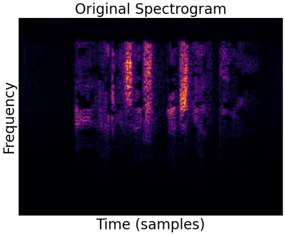


More on filtering

- Low-pass filtering (cut-off at 1kHz) Listen.



- High-pass filtering (cut-off at 4kHz) Listen.



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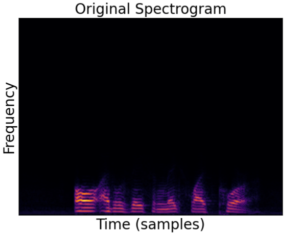

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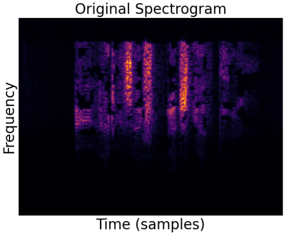


- There are also band-stop, band-pass filtering..
  - Il existe aussi band-stop, band-pass filtering.

Fast Convolution

- Convolution is an $\mathcal{O}(N^2)$ operation.
- However we can use FFT to get it down to $\mathcal{O}(N \log N)$.
  - On peut prendre avantage de le FFT pour avoir une complexité de $\mathcal{O}(N \log N)$.
- Convolution in time domain, is multiplication in the Fourier Domain.
  - Convolution dans le domaine de temps est multiplication dans le domaine de Fourier.

$$
\begin{aligned}
F(x * w) =& Fx \odot Fw \\
\rightarrow x * w =& F^{-1}(Fx \odot Fw)
\end{aligned}
$$

- Use FFT whenever you can!

Convolution as a Matrix Multiplication

- Convolution can be expressed as a matrix multiplication.
  - Convolution peut-etre exprimée comme une multiplication de matrices.

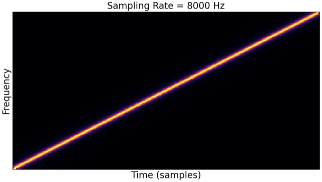
$$w * x = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \underbrace{\begin{bmatrix} w_1 & 0 & 0 & 0 \\ w_2 & w_1 & 0 & 0 \\ 0 & w_2 & w_1 & 0 \\ 0 & 0 & w_2 & w_1 \\ 0 & 0 & 0 & w_2 \end{bmatrix}}_{:=C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

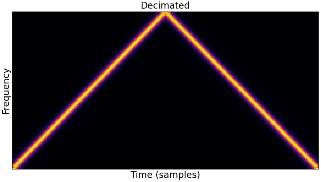
- C is a Circulant / Toeplitz matrix.

Table of Contents

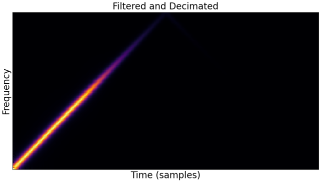
Downsampling

- Let's say we want to downsample. Simple decimation introduces aliasing.
- Si on sous-echantillonne, simple decimation introduit de l'aliasing.





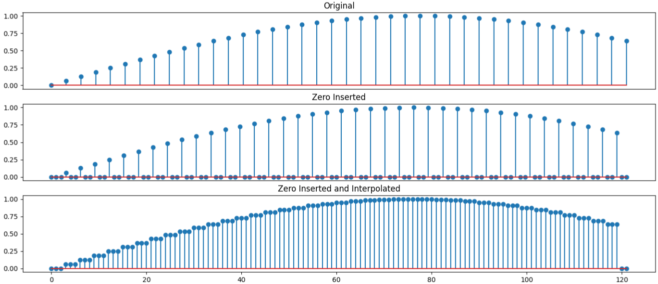
- The solution is to first filter and then downsample.
  - La solution est de filtrer et puis downsampler



- We lose the high-freqs, but we at least avoid aliasing.
  - On perd les hautes frequences mais on moins pas d'aliasing.

Upsampling

- To upsample by a factor of L, the procedure is to first insert $L - 1$ zeros, and then to interpolate.
  - Pour upsampler de la facteur L, le procedure est de d'abord inserer $L - 1$ zéros, et puis faire de l'interpolation.



Recap

- Signal Representations
  - Time, Frequency, Time-Frequency
- Discrete Fourier Transform
- Short-Time Fourier Transform
- Sampling, Resampling
- Convolution

Furher Reading

- http://www.dspguide.com/pdfbook.htm – nice free book, check it out.
- https://dspguru.com/dsp/howtos/

Next week

- We get started with machine learning (sometimes for signal processing)